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ABSTRACT

Abstracts of 36 research reports are provided. The reports were prepared by investigators for presentation at the 55th annual meeting of the National Council of Teachers of Mathematics. A broad range of topics related to mathematics education is covered. Nine reports deal with problem solving, eight are concerned with instructional methods, five with space and geometry, and four with numbers and operations. Two reports concern reading and writing skills in mathematics, two deal with testing and measurement procedures, and two concern program evaluation. Other papers deal with logic, effective teachers, learning aids for the blind, and models of mathematics learning. (DT)

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MATHEMATICS EDUCATION REPORTS

RESEARCH SECTIONS
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
55th ANNUAL MEETING

Cincinnati, Ohio
April 20 - 23, 1977

ERIC Center for Science, Mathematics,
and Environmental Education
College of Education
The Ohio State University
1200 Chambers Road, Third Floor
Columbus, Ohio 43212

December, 1976

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PREFACE

The ERIC Center for Science, Mathematics, and Environmental Education is pleased to make available this compilation of abstracts of the Research Sections of the 55th Annual Meeting of the National Council of Teachers of Mathematics. Readers of past compilations of papers from Research Reporting Sections will want to note that this compilation not only contains such papers, but also contains abstracts of panel presentations and round table discussions.

Round table discussions are an experimental format being tried for the first time at the 1977 meeting. These sessions provide interested persons an opportunity to discuss in depth a specific piece of empirical research with the researcher(s). There will be no presider at the round table discussion sessions. Only one paper will be presented and the author(s) will be on hand to discuss the research, answer questions and interact with participants during the scheduled time. Importantly, these sessions are not designed for large audiences. Each room will be equipped with a single table surrounded by chairs to facilitate an informal, small group discussion. Participants interested in a specific round table discussion are urged to be at the assigned room early, since seating will be limited.

I wish to express my appreciation to the program chairman for the research sections, Mark Spikell. Dr. Spikell was responsible for the original Call for Papers, and chaired the selection process. Nick Branca, Don Dessart, Terry Coburn, Richard Shumway and Les Steffe, as members of the NCTM Research Advisory Committee, served as the selection panel. Selection of these papers and presentations was made from over 114 submissions.

December, 1976

Jon L. Higgins, editor
Associate Director for
Mathematics Education

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Thursday, April 21, 1977 Panel Presentation
10:30 - 11:30 a.m.

TASK VARIABLES IN PROBLEM SOLVING RESEARCH

Gerald Goldin
University of Pennsylvania

Ed McClintock
Florida International University

Norman Webb
Indiana University

Purpose

This session will review current research issues pertaining to the role of task variables in mathematical problem solving and discuss the National Collection of Research Instruments for Mathematical Problem Solving. The participants are members of an editorial board that is developing the Collection for the use and reference of problem solving researchers and teachers.

Rationale

An important component of problem solving research in mathematics is the selection of the problems that are used in the treatment phase or in the evaluation phases of the study. Similarly, an important aspect of teaching is the selection of problems that develop students' ability to solve problems. Teachers and researchers frequently select or develop problems which have the appropriate context, content, and level of complexity for the subjects, as well as addressing the intended teaching and research variables. If the use of problem solving heuristics is of interest, for example, it may be that certain problems are capable of eliciting particular strategies or approaches, while for other problems these are inapplicable or irrelevant. Typically, the teacher and the researcher are interested in controlling for some characteristics of problems, and in varying others. Thus it is crucial that information on the task variables be as complete as possible, and widely shared.

A working group of the Problem Solving Project of the Georgia Center for the Study of Learning and Teaching Mathematics has developed a set of categories for describing the task variables of mathematical problem solving instruments. Individual and cooperative research studies by the group have produced several sets of problems on which a wealth of task variable data is available, ranging from data on readability to data on problem-solving strategies used by subjects, and including analyses of problem state-space structures and problem symmetry.

These problem instruments are the first in the newly formed National Collection of Research Instruments for Mathematical Problem Solving. The problem instruments along with their associated data will be available to interested teachers and researchers; they, in turn are asked to submit problems and data to develop the Collection and to encourage cooperative efforts.

Problem task variables will be explained and illustrated, using problem instruments on which cooperative efforts have made available a substantial amount of task variable information. The National Collection of Research Instruments for Mathematical Problem Solving will be discussed, and the procedures for submitting and obtaining instruments and their documentation will be outlined. The proceedings of the session, and brief synopses of the content of the presentations are as follows:

1. Introduction

The definition, importance, and application of task variable description in problem solving research. The relationship between task variables and subject and treatment variables.

2. Heuristic Behavior Variables

The definition of heuristic behaviors and a discussion of how problems may be characterized by the heuristic behaviors employed in their solution.

3. Content and Context Variables

The definition of content and context variables and a discussion of how these task variables apply to certain problems. An examination of the implications of controlling and varying content and context with respect to subject and treatment variables.

4. Structure and Complexity Variables

The definition of structure and complexity variables and a discussion of examples of these variables in relation to sample problems. The importance of structure and complexity as independent variables in problem solving research and teaching will be explored.

5. The National Collection of Research Instruments for Mathematical Problem Solving

A description of the Collection and its operation will be given. The procedures for task variable documentation will be described, as well as procedures for submission of problem data. The audience will be provided with a catalogue describing the available instruments and how to obtain them.

Thursday, April 21, 1977
12:00 - 1:00 p.m.

Round Table Discussion

INFORMATION MEASURES OF PROBLEM COMPREHENSION AND SOLUTION

Harold Days
Purdue University

Purpose

The purpose of the study was to develop information content measures for two sets of problems. Each set consisted of five problems; a "target" problem and four others related to it in each of these ways: (1) equivalent, (2) similar (3) special case, and (4) general case (Wickelgren, Ch. 9). One set of problems was verbal with algebraic solutions and the other set required reasoning, with sequences of moves as solutions.

One of the greatest difficulties in studying problem-solving behavior is to determine what a subject is thinking during the solution of a problem.

Information analysis offers an alternative to the "think aloud" approach to the study of problem-solving behavior and to the study of techniques for improving problem-solving performance. The information of solving a related problem would make it possible to select sets of example problems for effective teaching of problem solving.

Conceptual Framework

The development of a theory of information by Shannon (1949) made it possible to consider a variety of psychological constructs from the viewpoint of information transmission. The "guessing-game" technique in which subjects guess each succeeding sign in a message produces a measure of information which accounts for the semantic and syntactic constraints in a message. The familiar expression in information theory which gives the information of a message with K signs is

$$I = -\sum_{i=1}^{k} \log_2 p_i,$$

where p_i is the probability of the ith sign. The probability of a sign can be determined empirically as the ratio of correct to total guesses for the sign over a sample of subjects.

This approach to information content gives the subjective information of a message as opposed to the statistical information which is computed using probabilities obtained from the distributional statistics of the signs. The subjective information of a message depends on the relationship between the format in which the message is communicated (eg. written, pictorial, oral, symbolic, etc.) and the characteristics of the receiver of the message (eg. age, aptitude, motivation, attitude,

etc.). Because of this relationship, subjective information has potential as a variable in studying cognitive processes.

Although it is useful in some applications to know the amount subjective information of a particular message for a certain population, it is usually of greater interest to know how much of that information has been transmitted by a teaching-learning process. For this reason, the concept of "transinformation" is especially relevant to education. Weltner (1973) has defined transinformation as follows:

The transinformation of a field X onto a field Y is

$$T(X,Y) = I(Y) - I(Y|X).$$

The expression $I(Y|X)$ is the "conditional" subjective information of a field Y, given field X. In this study the transinformation of a related problem onto a target problem was investigated.

The concept of transinformation has a second significant application. Any message contains both syntactic and semantic information. Transinformation analysis can be used to separate these components. For a subject who has solved a problem P, the only new information in it is the syntactic information, ie. $I(P\ P)$ is the syntactic information in P. Therefore,

$$T(P,\ P) = I(P) - I(P|P)$$

is the semantic information in the problem.

Procedures

Two sets of five problems each were prepared, as described below:

Set One

Problem	Description
IM(Target)	Cannibals and Missionaries problem (Wickelgren, p. 86)
2M(Equivalent)	Substitute outlaws and sheriffs in target problem
3M(Similar)	Fox, goose, corn problem (Wickelgren, p. 154)
4M(Special case)	Two missionaries and cannibals instead of three
5M(General case)	Add the constraint that only one cannibal can row the boat

Set Two

Problem	Description
IV(Target)	Polya's Hens and Rabbits Problem
2V(Equivalent)	Change hens and rabbits to boys and dogs
3V(Similar)	Change to nickels and dimes; different numbers
4V(Special case)	Number of hens = Number of rabbits = one
5V(General case)	Number of hens = h, number of rabbits = r

Forty-seven subjects in a college geometry course and in an algebra and trigonometry course were randomly divided into six groups, each group solved one of the five problems with the sixth group solving an unrelated problem. After each subject had solved the assigned problem, all subjects guessed the statement and solution of the target problem. The procedure was to project the problem on a screen and reveal one word at a time after subjects had written their guesses.

Analysis and Results

The analysis for problem set one has been completed. The subjective information (I) per word for the problem statement and for each of the ten steps in the solution was calculated for each group. The values are presented in Tables 1 and 2.

Table 1

Subjective Information per Word (Bits) of Target Problem Statement for Each Treatment Group

Group	Information/Word	[b]Transinformation onto 1M
1M (Target)	.72	2.92
2M (Equivalent)	2.94	.70
3M (Similar)	3.00	.64
4M (Special case)	.70	2.94
5M (General case)	[a] ___	___
6M (Unrelated)	3.64	

[a] Several subjects did not finish the guessing test, so data is not available.

[b] Calculated by subtracting each information/word from the total information, 6M.

Table 2

Subjective Information per Solution Step of Target Problem for Treatment Groups

Group	N	Solution Step 1	2	3	4	5	6	7	8	9	10	Total[a]	Transinformation
1M	9	0	.14	.14	.58	.58	.14	.58	.14	.14	.14	2.58	.97
2M	8	0	0	.38	.26	0	0	.26	0	0	0	.90	2.65
3M	8	.38	.17	.69	.26	0	0	.38	0	.26	0	2.11	1.44
4M	8	0	.17	0	0	0	.17	.17	0	0	.17	.68	2.87
5M	5	0	0	0	0	0	0	.76	.28	.28	.28	1.60	1.95
6M	9	.14	.38	1.15	.38	.14	.58	0	.38	.14	.14	3.55	____
Total[b]	47	.13	.20	.37	.26	.15	.20	.24	.15	.13	.15	1.98	3.55

[a] computed by summing information in all steps.

[b] computed by calculating p_i for all subjects.

Conclusions

The information for group 1M can be interpreted as syntactic information due simply to language structures that are not recalled word-for-word. The transinformation values in Table 1 represent the semantic information of the problem statement that was gained by each group. Group 4M was the only group other than the target group in which substantial information was gained.

Two important inferences about influences on problem difficulty can be made from the data of Table 2. First it can be seen that certain steps in the solution contain more information than others. For example, steps 3, 4, 6 and 7 are somewhat more difficult than others. An examination of the state-space structure of the problem reveals that there are more potential exits from states (steps) 3 and 6 than any others and step 6 involves a detour "backwards" to a lower level, accounting for more information content. Although they do not have high state-space information content, steps 4 and 7 are made more difficult, subjectively, due to a "carry-over" or interference from a previous difficult step.

Finally, the total information for each group can be used to compute transinformation values as shown in the last column of Table 2. The greatest information gain due to solving related problems was for groups 2M and 4M, the equivalent and special cases. The surprisingly low value for 1M may be explained by the interference of a forced syntax for recording solutions during the experimental procedure.

Further analysis, comparing subjective and state-space information content appears to hold promise for constructing a model of problem solving difficulty and processing. This work, along with results related to algebra verbal problems will be reported.

Thursday, April 21, 1977
12:00 - 1:00 p.m.

Panel Presentation

THE USE OF GAMES IN MATHEMATICS INSTRUCTION: A REVIEW, CRITIQUE, AND INTERPRETATION OF THE LITERATURE AND A PROPOSAL FOR FUTURE INVESTIGATIONS

George W. Bright
Northern Illinois University

John G. Harvey
University of Wisconsin - Madison

Margariete Montague Wheeler
Northern Illinois University

This session will provide synthesis of the available literature on the cognitive effects of games used in teaching mathematics and suggest a program of research for systematically investigating unsolved problems related to the use of games in the classroom. The synthesis is needed because the literature on the cognitive effects of games is extremely fragmented.

This literature includes the largely unpublished work of the Center for Social Organization of Schools at the Johns Hopkins University, studies employing games in conjunction with advance organizers, and the evaluation of curriculum materials incorporating games as part of instructional strategies. Also, Dienes (1973) has used games in teaching mathematical structure, and Branca and Kilpatrick (1972) and Goldin and Luger (1973) have studied problem solving strategies in game situations. (Piaget's studies on the development of logical analysis would seem to be relevant to the analysis of the acquisition of game strategies.) Finally, TORQUE (a project at Education Development Center, Inc., 1976) proposes to use games in measuring achievement. Nowhere, however, has there been an attempt to correlate the results of these varied endeavors.

There are several obvious problems in trying to provide a synthesis of this literature. First, there is no common theoretical base. For example, Goldin and Luger (1973) used the perspective of artificial intelligence, Piaget's work is based on genetic psychology, and the Johns Hopkins project evolved from extensive studies of sociological variables. Second, there seems to be little agreement not only on the important cognitive variables to be measured but also on the procedures for measuring cognitive variables. Third, the uses to which games have been put run the spectrum from the introduction of concepts (Scandura and Wells 1967) to the practice of skills (Edwards and DeVries 1972).

This session will assist in the development of a perspective of the status of current knowledge of the use of games in mathematics and will suggest ways for obtaining more information on critical questions regarding the use of games to improve mathematics learning. This

perspective is needed in part because teachers seem to be constantly bombarded with claims that games are in fact effective tools for teaching and maintaining concepts and skills. (For example, in 1973 and 1974 there were more than 15 articles describing mathematics games published in the Arithmetic Teacher.) Yet, very little seems to be known about the extent of cognitive effects of games.

Additionally to this purpose it is hoped that other mathematics educators will become interested in studying the effects of games. There is at least the potential for organizing a future presession on games for the Special Interest Group/Research in Mathematics Education of the American Educational Research Association.

The presentation will be made through a panel composed of the three proposers. A reactor will be invited to respond to the questions: (1) What are the most important, currently verified effects of games in the context of the entire scope of mathematics education? and (2) What questions relative to the effects of games need to be answered most urgently? Some time will be set aside for questions and reactions from the audience.

The presentation will be structured in several sections:

1. Games, their common characteristics, and ways of classifying them

2. Research related to significant kinds of games and uses of games

   a. Simulation games: structural characteristics (e.g., team versus individual competition) as related to cognitive variables; structural variables as studied in the Johns Hopkins University project

   b. Teams - games - tournament (TGT) model: description of the TGT model; results of use of TGT across grade level; cognitive effects of modifications of the TGT model

   c. Other uses of games: games and advance organizers (Scandura and Wells 1967, Lesh and Johnson 1976); games and mathematical structure (Goldin and Luger 1973, Branca and Kilpatrick 1972, Dienes 1973); curriculum projects incorporating games (using games to introduce content, to develop content, to promote retention of concepts, and to promote mastery of skills)

3. Measurement of variables in game settings: cognitive effects of games; using games to measure cognitive variables; acquisition of game strategies and the effects of acquisition on cognitive variables; the development of logical analysis as related to acquisition of game strategies.

4. Synthesis of the available information: identifying strengths and deficiencies in the research base; stating and classifying the most critical unresolved research questions; suggesting appropriate means of obtaining answers to these questions.

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Thursday, April 21, 1976
1:30 - 2:30 p.m.

Round Table Discussion

1965-1975: ACHIEVEMENT AND ANALYSIS OF COMPUTATION SKILLS TEN YEARS LATER

Ann D. Hungerman
The University of Michigan

Purpose

This research study was undertaken for the purpose of answering two different but related questions. The first, a substantive one, "Do the computation skills of sixth grade pupils in a southeastern Michigan school district in 1975 differ significantly from those of a similar group a decade earlier in 1965?" The second, a procedural one, "Can achievement of computation skills be more accurately measured and reported with an altered and expanded research design, utilizing statistical procedures which are available in 1975, but which were not widely used in 1965?" Specifically, what effect upon results will be evident if the unit is defined as a class rather than as an individual, and if multivariate and/or pattern analysis is employed rather than analysis of variance and covariance?

Conceptual Framework

The following assumptions provide a framework of meaning for the collection and analysis of data describing computation skills of sixth grade pupils.

1. Computation skill is a major goal, but not the only goal of elementary mathematics instruction.

2. A high level of computation skill should be maintained in any sound elementary mathematics curriculum.

3. Computation skill is influenced by many variables. Student ability, program, method, and teacher are several of the most frequently acknowledged influences.

4. The current criticisms of "modern math" admittedly have some basis in reported lower achievement scores, but these may be over generalized or inaccurately interpreted.

5. The rationale for the expansion of the research design can be found in the description of newer statistical procedures which attempt to improve analysis of related variables by examining them as components of an interrelated group, rather than as a series of simple and separate variables.

Procedures

The California Arithmetic Test (Part II - Fundamentals (80 items)) was administered to sixth graders in ten schools at the end of the school year, 1975, by the reporter. The same test had been given in the same district at the same time in 1965. School records were the source of descriptive variable data: age, sex, and I.Q. - total, language, and non-language. (Intelligence data were derived from the California Test of Mental Maturity in 1965, and from the California Short Form Test of Academic Aptitude in 1975.) The mathematics coordinator was interviewed and the sixth grade teachers surveyed to determine the prevailing organizational practices, the methods and textbooks in most common use.

Analysis

Analysis of variance (ANOVAR) and analysis of covariance (COVAR) with the three I.Q. scores as covariates were employed to measure mean differences in total computation, addition, subtraction, multiplication, and division. A histogram program revealed the percentage of students correct on each of the 80 items in 1975. These data were then compared to similar 1965 data. (Data summaries are attached.) The above constitutes a replication of the computation section of the 1965 study. The expansion of the research design to include multivariate and/or pattern analysis will commence in May, 1976, and should be completed during the summer, 1976. Differences in percentage correct data will be tested for significance also during this time.

Results and Conclusions

This section is prefaced with the comment that the findings vary somewhat with the statistical procedure being followed, and the caution that conclusions which seem apparent from initial analysis are refined as subsequent, more specific analyses are employed. The 1965 and 1975 groups did not differ significantly in total I.Q. means.

Findings:

1. There is no significant difference between the 1965 and 1975 groups in total computation revealed by analysis of covariance with either total I.Q. or language I.Q. as covariate. The use of non-language I.Q. as covariate alters this result to favor the 1965 group, and calls attention to an interesting but atypical pattern of intelligence data evidenced by the 1975 group - a higher non-language I.Q. than total or language I.Q.

2. The 1965 group is significantly favored in addition and subtraction, when ANOVAR or COVAR with any one of the three I.Q. scores as covariates is employed.

3. There is no significant difference between the 1965 and 1975 groups in multiplication revealed by any analysis.

4. The 1975 group is significantly favored in division at the .04 level when COVAR (total I.Q.) is used, and at the .007 level when COVAR (language I.Q.) is used. There is no significant difference between the 1965 and 1975 groups when ANOVAR and COVAR (non-language I.Q.) are used.

5. For individual items of computation, the 1975 group scored higher than the 1965 group on 10 to 20 addition items; 11 of 20 subtraction items; 13 of 20 multiplication items; and 16 of 20 division items. (Note: The 1965 group mean for addition and subtraction was significantly higher than that of the 1975 group.)

6. For individual items of computation, the 1975 group scored higher than the 1965 group on all 33 whole number items, but scored lower than the 1965 group on 20 of 30 fraction items, and on 5 of the 8 decimal items. (Note: The 1975 group matched the 1965 group in fraction/division items, and surpassed them in decimal/division items.)

Conclusions will be more appropriately and accurately drawn when the additional data analyses have been completed, and will be related to informally collected data describing text use and classroom organizational practices.

COMPUTATION SKILLS - Grade 6 - 1965 and 1975

California Arithmetic Test: Fundamentals

Group 1 - 1965, N=305, Ten schools
Group 3 - 1975, N=386, Ten schools

VARIABLE	GROUP	ANOVAR. MEAN	DIFF.	SIG.	S.D.
Total I.Q.	1	102.76	.16	NS	14.76
	3	102.92			12.31
Non-Language I.Q.	1	101.45	1.63	NS	15.46
	3	103.08			12.71
Language I.Q.	1	103.58	1.35	NS	14.47
	3	102.23			13.12
Computation Total (80 items)	1	44.131	1.064	NS	
	3	43.067			
Addition (20 items)	1*	12.751	.699	.01	
	3	12.052			
Subtraction (20 items)	1*	11.908	.696	.01	
	3	11.212			
Multiplication (20 items)	1	10.046	.147	NS	
	3	9.899			
Division (20 items)	1	9.4098	.494	NS (.0696)	
	3*	9.9042			

COMPUTATION SKILLS - Grade 6 - 1965 and 1975

California Arithmetic Test: Fundamentals

Group 1 - 1965, N=305, Ten schools
Group 3 - 1975, N=386, Ten schools

	VARIABLE	GROUP	ANOVAR. MEAN	COVAR. ADJ. MEAN	DIFF.	SIG.
COVARIATE: TOTAL IQ	Computation Total (80 items)	1 3	44.131 43.067	44.799 43.641	1.158	NS
	Addition (20 items)	1* 3	12.751 12.052	12.926 12.202	.724	.001
	Subtraction (20 items)	1* 3	11.908 11.212	12.088 11.367	.721	.001
	Multiplication (20 items)	1 3	10.046 9.899	10.204 10.035	.169	NS
	Division (20 items)	1 3*	9.4098 9.9042	9.563 10.036	.473	.040
COVARIATE: LANGUAGE IQ	Computation Total (80 items)	1 3	44.131 43.067	44.383 44.001	.382	NS
	Addition (20 items)	1* 3	12.751 12.052	12.817 12.295	.522	.026
	Subtraction (20 items)	1* 3	11.908 11.212	11.976 11.462	.514	.028
	Multiplication (20 items)	1 3	10.046 9.899	10.107 10.125	.018	NS
	Division (20 items)	1 3*	9.4098 9.9042	9.467 10.116	.649	.007
COVARIATE: NON-LANG. IQ	Computation Total (80 items)	1* 3	44.131 43.067	44.965 43.035	1.930	.009
	Addition (20 items)	1* 3	12.751 12.052	12.970 12.043	.927	.000
	Subtraction (20 items)	1* 3	11.908 11.212	12.134 11.204	.930	.000
	Multiplication (20 items)	1 3	10.046 9.899	10.238 9.891	.347	NS
	Division (20 items)	1 3	9.4098 9.9042	9.604 9.896	.292	NS

*Significant difference favors this group.

THE NUMBER OF STUDENTS CORRECT ON 80 INDIVIDUAL ITEMS OF COMPUTATION, EXPRESSED AS A PERCENTAGE OF THE GROUP, 1965 and 1975

ADDITION					SUBTRACTION				
	% Correct		Difference			% Correct		Difference	
Item Type	1965	1975	Sign	Size	Item Type	1965	1975	Sign	Size
1. WN	89.9	97.4	+	7.5	1. WN	85.9	93.3	+	7.4
2. WN	91.4	97.9	+	6.5	2. WN	88.5	94.3	+	5.8
3. WN	91.2	96.1	+	4.9	3. WN	82.7	91.7	+	9.0
4. WN	87.7	89.4	+	1.7	4. WN	87.7	93.0	+	5.3
5. WN	82.7	92.5	+	9.8	5. WN	79.4	90.4	+	11.0
6. WN	92.4	96.4	+	4.0	6. WN	67.2	71.8	+	4.6
7. WN	73.2	79.0	+	5.8	7. WN	56.5	69.2	+	12.7
8. Mon	59.9	71.8	+	11.9	8. Mon	56.0	64.8	+	8.8
9. Mon	33.2	30.8	-	2.4	9. Mon	33.5	26.9	-	6.6
10. RNF	71.3	66.6	-	5.2	10. RNF	78.8	79.0	+	0.2
11. RNF	55.3	47.9	-	7.4	11. RNF	74.6	75.9	+	1.3
12. RNF	75.8	80.6	+	4.8	12. RNF	50.0	49.7	-	0.3
13. RNF	57.4	51.8	-	5.6	13. RNF	44.8	44.3	-	0.5
14. RNF	53.2	45.3	-	7.9	14. RNF	75.7	78.5	+	2.8
15. RNF	45.4	43.5	-	1.9	15. RNF	26.1	8.8	-	17.3
16. RNF	35.0	33.2	-	1.8	16. RNF	34.9	24.9	-	10.0
17. RNF & DEC	25.6	19.7	-	5.9	17. RNF & DEC	26.8	14.8	-	12.0
18. DEC	37.1	21.2	-	15.9	18. DEC	25.5	16.8	-	8.7
19. DEC	27.7	19.9	-	7.8	19. DEC	29.2	19.7	-	9.5
20. DEN	22.6	26.9	+	4.3	20. DEN	23.7	13.7	-	10.0

KEY:

WN	Whole Numbers
Mon	Money
RNF	Rational Number/Fraction
DEC	Decimal
DEN	Denominate

THE NUMBER OF STUDENTS CORRECT ON 80 INDIVIDUAL ITEMS OF COMPUTATION, EXPRESSED AS A PERCENTAGE OF THE GROUP, 1965 and 1975

MULTIPLICATION					DIVISION				
	% Correct		Difference			% Correct		Difference	
Item Type	1965	1975	Sign	Size	Item Type	1965	1975	Sign	Size
1. WN	87.0	91.5	+	4.5	1. WN	84.9	91.7	+	6.8
2. WN	88.0	91.2	+	3.2	2. WN	85.3	93.5	+	8.2
3. WN	77.2	83.9	+	6.7	3. WN	84.9	88.9	+	4.0
4. WN	81.9	85.2	+	3.3	4. WN	64.6	71.8	+	7.2
5. WN	66.8	73.3	+	6.5	5. WN	65.1	72.5	+	7.4
6. WN	53.5	69.7	+	16.2	6. WN	42.5	44.8	+	2.3
7. WN	59.2	68.9	+	9.7	7. WN	68.1	76.9	+	8.8
8. WN	55.4	65.8	+	10.4	8. WN	30.3	44.6	+	14.3
9. WN	41.3	46.1	+	4.8	9. WN	51.0	54.4	+	3.4
10. RNF	56.7	44.0	-	12.7	10. WN	25.3	30.6	+	5.3
11. RNF	46.5	44.8	-	1.7	11. RNF	41.2	36.0	-	5.2
12. RNF	42.8	38.1	-	4.7	12. RNF	22.3	26.2	+	3.9
13. RNF	28.3	30.8	+	2.5	13. RNF	18.9	24.4	+	5.5
14. RNF	45.5	28.8	-	16.7	14. RNF	61.0	57.3	-	3.7
15. RNF	21.4	14.5	-	6.9	15. RNF	26.5	33.4	+	6.9
16. RNF	18.8	13.7	-	5.1	16. RNF	19.3	18.9	-	0.4
17. RNF	10.2	11.9	+	1.7	17. RNF	20.5	17.9	-	2.6
18. DEC	53.4	60.6	+	7.2	18. RNF	29.9	34.2	+	4.3
19. DEC	13.9	9.8	-	4.1	19. DEC	34.8	43.5	+	8.7
20. DEN	13.2	18.1	+	4.9	20. DEC	13.7	30.3	+	16.6

KEY:

WN	Whole Numbers
Mon	Money
RNF	Rational Number/Fraction
DEC	Decimal
DEN	Denominate

Thursday, April 21, 1977
1:30 - 2:30 p.m.

Research Reporting Session
Paper One

AN INVESTIGATION OF SELECTED VISUAL ABILITIES AND THEIR ROLE IN THE MATHEMATICS EDUCATION OF PRESERVICE ELEMENTARY SCHOOL TEACHERS

Charles Dietz
Northern Illinois University

Jeffrey C. Barnett
Northern Illinois University

Purpose:

The purpose of this study was to investigate the role of visualization on the ability of preservice elementary teachers to correctly perform a number of Piagetian tasks related to the mathematical education of children. In addition to providing a paradigm for future investigations of the relationship of visualization to instruction in mathematics, the study attempted to determine a quick, reliable method of diagnosing visualization difficulties that could influence the manner by which some students learn to use concrete materials.

Conceptual Framework

The ability to visualize and its relationship to physical, mathematical, and intellectual factors has been a topic of continuing research for many years. Correlational studies have attempted to assess the relationship of visualization to mathematical achievement in children, but few investigations have attended to the problem of determining how visual ability relates to particular tasks relevant to school mathematics. While it is evident that visualization plays an important role in the mathematical development of children, the assessment of visual abilities of preservice elementary school teachers has received little attention.

In recent years the use of manipulative materials for teaching mathematics in the elementary grades has achieved considerable popularity as an approach suggested in mathematics methods courses, texts, and laboratory manuals. An analysis of the abilities needed for preservice elementary teachers to learn through the use of manipulatives suggests that visualization ability is an important factor which becomes even more significant when both enactive (concrete) and iconic (pictorial representational) levels of abstraction are used extensively. Observations of preservice elementary teachers reveal a wide range of visualization ability levels, that are often manifested during laboratory activities. It would seem from Piagetian research that by the time students reach the age of formal operations (approximately 12 years old), concepts such as vertical and horizontal reference systems should be well developed. Yet, pilot studies conducted by the investigators have shown that a surprising number of preservice elementary teachers (ages 18 to 22) failed such

Piagetian tasks as the "bottle problem". (Students are shown a picture of a jug about one third full of water, resting on a flat, level surface. They are then shown a picture of the same jug tilted on a 45 degree angle, and asked to draw in where they think the water line will be.)

Procedures

The subjects for the study consisted of approximately 250 preservice elementary school teachers enrolled in either an elementary mathematics methods course or in a prerequisite mathematics content course, during the Spring term, 1976. During regularly scheduled classes, students were given a standardized visualization test involving the recognition of patterns in paper folding tasks. Following the visualization test, the "bottle task", box task, and faucet-bottle tasks were administered, one at a time. In the box task, students were shown a cut-away view of a square box with a pendulum suspended from the center. They were then shown a view of the same box tilted on a 45 degree angle and asked to draw in where they think the pendulum should be. The faucet-bottle task was used as a check on the bottle task. A picture of a tilted jug under a running faucet was shown to the students. They were then asked to draw in where they thought the water line would be when the jug was half full.

Correlations between the three tasks and the visualization scores were computed. Over 55 percent of the students in the study failed the bottle task, which correlated highly with the visualization scores. While the majority of the students performed correctly on the box task, those who performed incorrectly also performed incorrectly on the other two tasks and had very low visualization scores. It would seem that the concepts involved in the box task represent necessary but not sufficient conditions for understanding concepts found in the bottle tasks.

Following the in-class tests, 58 students were selected at random for detailed interviews to determine the reliability of the "bottle task" and to investigate some possible visualization factors that might explain student responses. Students were asked to repeat the bottle task in a number of forms. Those students who were consistently wrong in responding were shown an actual jug of water and asked to predict where the water line would be if it were tilted. Confronted with their own contradictory responses, students were either surprised that the water level was parallel to the table top, or continued to see the water level as non-parallel, even though they were looking at the real jug.

To test Piaget's contention that the development of vertical and horizontal reference systems are prerequisite to success on the "bottle task", students were shown a sheet of paper with four dots and asked to locate the dots in exactly the same places on another sheet of paper, using any method they wished. The majority of the students measured from the top and sides of the paper without hesitation. Those that failed this task were equally divided on the "bottle task" so further testing is needed before any conclusions can be made.

In the final phase of the interview, students were asked to state the number of faces, edges, vertices of wooden blocks of various shapes. Again, while most students responded correctly, those that did not also did not do well on all of the other tasks.

It is apparent from the data that visualization plays an important role in a variety of tasks related to the concrete operational stage of development. Student responses were found to be very consistent across tasks and correlated highly with visualization scores, as well as success in the methods course. More research is clearly needed to help determine the precise role of visual ability in the mathematics education of teachers. The data obtained in this study indicate that visualization could be an important factor in future curricular and instructional decisions.

Thursday, April 21, 1977
1:30 - 2:30 p.m.

Research Reporting Session
Paper Two

PRE-SERVICE ELEMENTARY TEACHERS' INTERPRETATIONS OF LOGICAL STATEMENTS IN THE CONTEXT OF ELEMENTARY MATHEMATICS

Suzanne K. Damarin
The Ohio State University

Purpose

The use of logic by pre-service elementary teachers has been the subject of several recent studies (Eisenberg and McGinty 1974, 1975; Jansson 1975; Juraschek 1976; Damarin 1976). These studies have been concerned with subjects' inferences from and interpretations of statements in the standard logical forms. Taken together they have shown that the population of pre-service elementary teachers does not use propositional logic accurately (if at all) when responding to items concerning a variety of situations which do not explicitly involve mathematics.

The purpose of the research to be discussed here was to begin an examination of pre-service elementary teachers' use of logic in the context of elementary mathematics.

Conceptual Framework

Use of the language of propositional logic is but one of several ways of stating logical relationships among mathematical variables. Three modes of presenting information concerning dichotomous concepts (such as odd/even, positive/negative, zero/nonzero) were identified. The context of odd and even counting numbers was selected for initial empirical work.

If M and N are two variables defined over the positive integers, information concerning the parities (oddness or evenness) of M and N can be presented in at least three ways: (1) a logical statement, that is, a simple statement or one involving a logical connective; (2) a mathematical statement involving an operation defined over the counting numbers; or (3) a sorting of the set of possible combinations of parities of M and N into a "truth set" and its complement. Examples of these presentations of relationships are given in table 1.

Table 1. Presentations of equivalent information in 3 modes

Mathematical Statement	Logical Statement	Sorting of the "replacement set"
M + N is odd	M is even if and only if N is odd	Truth set = (M odd, N even), (M even, N odd) ; Complement = (M odd, N odd), (M even, N even)

24/25

M x N is even	M is even or N is even (or) If M is odd then N is even	Truth set: (M odd, N even), (M even, N odd), (M even, N even)
M x N is odd	M is odd and N is odd	Truth set: (M odd, N odd)

The importance of logic to the teacher of elementary school mathematics would seem to lie in the ability to use all of these presentation modes, and to move freely from one to another in classroom discourse.

Procedures

The study was conducted in several phases. In each phase a test or tests was administered to a population of pre-service elementary teachers enrolled in a mathematics or mathematics methods course. The sample size for each phase was thus dependent upon the enrollment of the participating class. The purpose of each phase, and the number of subjects involved, is given in the table below.

Table 2. Purposes of each phase of the study

Phase	Purpose	N
1	Compare abilities of subjects to sort replacement set given statements in mathematical vs. logical form	29
2	Compare abilities of subjects to select logical vs. mathematical statements given a sorting of the replacement set	57
3	Test ability of subjects to select element of replacement set on which two statements differ; compare difficulty for both logical, both mathematical, and mixed statement forms	120
4	Repeat phase 1 and abbreviated phase 2; compare subject abilities on phase 1 and 2 tests with ability to translate from one mode (logical, mathematical) to the other	(est)150

For each of the 16 possible sortings of the "replacement set" (M odd, N odd), (M odd, N even), (M even, N odd), (M even, N even) a standard mathematical statement and a standard logical statement were determined. (For sortings which could be expressed in disjunctive or conditional forms two standard logical statements were generated and used.) These standard statements were used in the construction of all tests.

For each phase four tests were constructed and spiralled for administration. These tests were parallel in that the nature of the task was the same (e.g., sorting of replacement set, translating from one mode to another), and in that items presented the same information, but differed in the roles played by mathematical and logical statements. The general test development strategy was to identify two roles (A and B) for statements within the test; forms of the test were generated by varying the form of the statements in each role. The four test forms resulting are identified in table 3.

Table 3. Scheme for developing tests for each phase

Form of Role B Statements	Form of Role A statements	
	Mathematical	Logical
Mathematical	Test MM	Test LM
Logical	Test ML	Test LL

Analysis

Analysis of data has not yet been completed. Repeated measures analyses of variance will be performed on scores for subscales of each test to determine the effects of logical vs. mathematical statements, and of specific logical connectives or mathematical operations involved in statements. Regression analysis will be performed on phase 4 data to determine the relationship between success on sorting, selecting statements for sortings, and translating from one mode to the other.

Analysis of data collected in phase 1 indicated that sorting of the replacement set is more difficult for mathematical than logical statements when the replacement set contains general terms (e.g. (M is odd, N is odd)) ($p < .001$). When the replacement set contains specific values (e.g. (M = 1, N =3)) the sorting is equally difficult for logical and mathematical statements. In both cases the order of difficulty of items presenting equivalent information changes with the mode of the statement.

For phase 2 (and subsequent phases), the replacement set contained specific values for M (1 or 2), and general values for N (odd or even). Preliminary analysis of phase 2 data revealed that the overall difficulty of selecting statements descriptive of sortings is about the same for mathematical and logical statements, and that the order of item difficulty is approximately the same for the two modes.

Further analysis of phase 2 data and analysis of phase 3 data is currently underway. Administration and analysis of phase 4 tests is scheduled for early May.

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Thursday, April 21, 1977
1:30 - 2:30 p.m.

Research Reporting Session
Paper Three

THE INTERACTION OF FIELD INDEPENDENCE AND GENERAL REASONING ABILITY WITH INDUCTIVE INSTRUCTION IN MATHEMATICS

John T. Briggs
Mount Gravatt College (Australia)

Douglas B. McLeod
San Diego State University

Purpose

Research on mathematics teaching indicates that no one instructional treatment is likely to maximize learning for all students. Instead of looking for a single "best" treatment, Cronbach (1957) has suggested that different treatments should be provided for students having different aptitudes. Cronbach's suggestion has led to a substantial amount of research on aptitude-treatment interactions (ATIs); for recent reviews, see Berliner and Cahen (1973) and Cronbach (1975).

One variable of interest in ATI studies is cognitive style. A dimension of cognitive style that is currently receiving much attention is Witkin's notion of field-dependence-independence (Witkin, Moore, Goodenough, & Cox, in press). Field-dependence-independence is related to the ability to separate part of a field from the whole. This ability is assessed through a perceptual task, but it is also related to certain kinds of intellectual performance and social behavior. Field-independent students, for example, tend to do better in mathematics and science; also, they are more successful in imposing structure on an unstructured setting. On the other hand, field-dependent students are more adept at learning social material, while they seem to be handicapped by an unstructured learning situation (Witkin et al., 1975).

The differences in abilities and preferences between field-independent and field-dependent students are related to at least some aspects of discovery learning in mathematics. For example, Carpenter, McLeod, and Skvarcius (1976) found a significant disordinal interaction between field-dependence-independence and the level of guidance of instruction in mathematics. As predicted by the theory, field-independent students did significantly better when the treatment provided minimal guidance, while the field-dependent students seemed to learn more under conditions of maximal guidance.

Since field-dependence-independence interacts with the level of guidance, it seems likely that it will also be related to other dimensions of discovery learning, such as inductive and deductive approaches to mathematics. There has been a substantial amount of research on inductive (discovery) and deductive (expository) teaching, including a number of studies done in an ATI setting (e.g., Behr & Eastman, 1975).

While this research has not found any consistent pattern of ATIs in mathematics, the review by Berliner and Cahen (1973) suggests that inductive-deductive teaching strategies are worthy of further investigation in an ATI context.

ATI studies using inductive-deductive teaching strategies frequently include a measure of general reasoning as an aptitude variable. Eastman and Carry (1975), for example, conjectured that the significant interaction that they found might have been due to the relationship between general reasoning and inductive-deductive differences in their treatments. They found that students who scored high in general reasoning tended to do better in the more deductive "analytical" treatment, while students with low scores in general reasoning were likely to learn more in the inductive "graphical" treatment. Even though this conjecture was not supported in a later study (Behr & Eastman, 1975), it seemed appropriate to include general reasoning in this study.

In summary, the purpose of this study was to search for aptitude-treatment interactions between two aptitude variables and two treatments. The aptitude variables were field-dependence-independence and general reasoning, and the two treatments used either an inductive or a deductive approach to teaching mathematics.

Procedures

The 66 students who participated in the study were enrolled in the third semester of a four-course sequence for elementary school teachers. The students were assessed on both aptitude variables and randomly assigned to either an inductive or deductive instructional treatment. The Hidden Figures Test was used to measure field-dependence-independence, while general reasoning was assessed through the Necessary Arithmetic Operations test.

The topic chosen for instruction was mathematical relations. The materials presented the reflexive, symmetric, and transitive properties of equivalence relations in a concrete manner; the relations were defined on a set of four elements, where the elements of the set were colored rods of four different lengths. The use of a concrete model was designed to insure that the topic would be accessible to all students, regardless of their mathematical background.

Each student worked individually on a packet of printed materials that used a programmed format. Both of the treatments presented the same content, used the same problems, and provided the same amount of practice. There was a minimum of teacher-student interaction; students with questions were directed to the relevant examples and explanations in the printed materials.

The two treatments differed only in terms of sequence characteristics. In the inductive treatment, verbalization of the concepts was delayed until after the discussion of relevant examples. In the deductive treatment, the description of the concept came first and was followed by positive and negative instances of the concept and opportunities for practice. Students were given one hour to complete the treatments.

An achievement test was administered immediately after the completion of the treatments. It assessed the student's ability to apply the concepts to relations on finite sets. Two days later the transfer test was given; it asked the students to apply what they had learned about finite sets to relations on infinite sets. Four weeks after the completion of the treatments, the achievement and transfer tests were readministered as measures of retention.

Results

The results were analyzed using multiple regression techniques on each of the four dependent variables (Kerlinger and Pedhazur, 1973). There were no significant interactions between field-dependence-independence and treatments; in fact, only for the transfer test was there any support for the hypothesis that field-independent students would do better in an inductive treatment.

There were, however, two significant interactions between general reasoning and instructional treatment. On both the immediate achievement test and the transfer retention test, the students who scored high on general reasoning did better in a deductive treatment. These results are consonant with the conjecture of Eastman and Carry (1975). Data from the other two tests show similar trends in support of this hypothesis.

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Thursday, April 21, 1977
1:30 - 2:30 p.m.

Research Reporting Session
Paper Four

THE EFFECTS OF THE USE OF MANIPULATIVE MATERIALS ON STUDENT PERFORMANCE IN THE ENACTIVE AND ICONIC MODES

Phillip M. Eastman
Northern Illinois University

Jeffrey C. Barnett
Northern Illinois Univeristy

Purpose

In recent years the use of manipulative materials for teaching mathematics in the elementary grades has received considerable support from many components of the mathematics education community. Despite this growing interest in the use of these materials, the amount of research on the preparation of teachers to use manipulative aids is quite limited. In the absence of directives from empirical research, decisions concerning the role of manipulatives in the elementary mathematics methods course have been made on practitioners "hunches" or on assumptions of the effectiveness of the use of such materials. In view of the expense of manipulative aids and the relatively large amount of class time needed to provide prospective elementary teachers physical experiences with these materials, it is clear that the lack of an empirical basis for this decision-making process is unacceptable.

The purpose of this study was to submit to empirical test, the prevailing assumption that prospective elementary teachers need actual physical experience with manipulative materials to be able to demonstrate numerical and structural properties of the four basic arithmetic operations in the enactive (concrete) and iconic (pictorial representation) modes. Specifically, the study attempted to test the following hypotheses:

1. There are no significant differences in measures of ability to demonstrate numerical and structural properties of the four basic arithmetic operations, between subjects who are required to operate in the enactive and iconic modes, and subjects who are restricted to operate only in the iconic mode.

2. There are no significant differences in measures of mathematical achievement between subjects who are required to operate in the enactive and iconic modes, and subjects who are restricted to operate only in the iconic mode.

Procedures

The subjects for the study consisted of 78 elementary education majors assigned to two sections of an elementary mathematics methods course. One

of the two sections was randomly selected to be the experimental group, while the other was designated to be the control group. Both groups met twice each week for a one period lecture-discussion session, and once each week for a double period laboratory session.

The topics for instruction consisted of three laboratory units on whole number properties, properties of addition and subtraction, and properties of multiplication and division, taken from a laboratory manual authored by Dale Jungst. Each unit consisted of several demonstrations and examples, followed by a set of exercises which required the students to draw pictures of the manipulative devices to demonstrate arithmetic properties (commutativity, associativity, etc.). The experimental group was required to demonstrate the arithmetic properties on their desks, using the concrete materials, before they were permitted to complete each exercise in the laboratory manual. The control group was not provided with concrete materials and worked the laboratory exercises in the manual entirely in the iconic mode.

Following the third laboratory session, a unit test was administered to all subjects in both groups. The first part of the test was similar to the exercises found in the laboratory manual and required responses in the iconic mode. The second part of the test was an evaluation of arithmetic achievement. To assess student ability to physically demonstrate a property using a designated manipulative aid (enactive mode), 19 subjects were randomly selected to be interviewed by one of the investigators. Each of these students was presented four properties and required to demonstrate them using centimeter rods, pegboards, set demonstration objects, and centimeter grid paper. Each demonstration was scored according to a point scale which gave partial credit for each correct step.

Analysis

The study employed a pretest-posttest control group design. Since random assignment of subjects to the two groups was not possible, a 40-item mathematical achievement test was used as a covariate for each of the dependent variables. The hypotheses were tested using a linear regression model. The alpha level of rejection for each statistical test was .05.

No significant differences were found between the two groups on the laboratory portion of the posttest or on the interview scores. Hypothesis 1 was therefore not rejected for student responses in either the enactive or iconic modes. Significant differences were found favoring the experimental group on the mathematical content portion of the posttest. Hypothesis 2 was therefore rejected.

Conclusions

The results of the study support the contention that it may not be necessary for preservice elementary teachers to have physical experience with manipulative aids in order to be able to use them to demonstrate

arithmetic properties. Not only were no significant differences found between the groups on the laboratory portion of the posttest and interviews, but the control group completed the laboratory exercises in approximately 25% less time than it took the experimental group. Extended over the course of a semester, this result could mean that more material could be covered in the same time period without loss of ability to use manipulative aids. However, in view of the better performance by the experimental group on the mathematics content portion of the posttest, it would seem that the physical manipulation of concrete materials can be effective in promoting and reinforcing greater understanding of mathematics principles.

Based on this investigation, it is apparent that the relative merits of economy of time and increased learning must be weighed carefully when formulating instructional decisions regarding concrete materials. The hypotheses proposed in the study clearly merit further investigation before a sound basis for these decisions can be established.

Thursday, April 21, 1977
3:00 - 4:00 p.m.

Round Table Discussion

AN ATTEMPT TO DESIGN INSTRUCTIONAL METHODS IN MATHEMATICS TO ACCOMMODATE DIFFERENT PATTERNS OF APTITUDE

Jerry P. Becker
Northern Illinois University

Courtney D. Young, Jr.
Nanuet High School (New York)

Purpose

The purpose of this research was to determine whether two instructional treatments ("Guided Discovery" and "Meaningful Didactic") would interact with two aptitude variables ("Reading" and "Reasoning"). Many researchers continue to pursue work aimed at acquiring a better understanding of students' ability to learn from instruction. Consistent with this, the present study looks at the question of how students with different aptitude patterns respond to different instructional techniques. As such, the study contributes towards the body of research which may show the way for acquiring additional knowledge and a theory that reflects the manner in which aptitude variables are differentially related to learner performance under various methods of instruction. The study falls in the area of research generally termed ATI research. The design and methodology employed in this study is different from that conventionally used by researchers, and is one that the authors believe should be used more widely by researchers.

Conceptual Framework

The conceptual framework has already been set forth in papers, for example, by Cronbach (1957), Cronbach and Snow (1969), and Becker (1970), in the American Psychologist, USOE Final Report, and the Journal For Research in Mathematics Education, respectively.

Procedures

1. Treatments: The programmed learning treatments were concerned with devising formulas for finding the sum of the first in terms of various number series.

2. Aptitude variables: Two main aptitude variables were involved -- "reading" and "reasoning." In addition, data from four other "predictor" variables were used in the regression analyses.

3. Outcome measures: Three outcome measures were used: "Initial Learning" measured recall of terms, notation, and formulas; "Formula Derivation" measured subjects' facility in constructing or deriving formulas for summing various number series; "Pattern Derivation"

measured subjects' facility in deriving the n-th term formula for various number series.

4. Subjects: Subjects were 62 eighth-grade students, selected from a large 'pretest' population (several hundred students). Subjects were selected from a bivariate distribution, who exhibited combinations of "High-High", "High-Low", "Low-High", and "Low-Low" scores on the Reasoning and Reading aptitude measures. From these groups, then, 31 "matched pairs" of subjects were identified. Subsequently, subjects in each matched pair were randomly assigned to the two experimental treatments—one took the "Guided Discovery" treatment, and the other took the "Meaningful Didactic" treatment.

5. Experimental Method: Subjects first worked through an introductory program in order to learn the terminology and symbols used in the treatments. Subsequently, subjects worked through their assigned treatment. After completing their treatments, subjects were administered three outcome measures, with short rest periods after the first and second ones.

Analysis

An ANOVA was used to examine data from the aptitude, predictor, and outcome variables. Then regression analyses were made of data to determine whether any interactions (ordinal or disordinal) existed. The statistical procedure used is one presented by Gujarati (1910), and involves generation of a dummy variable, which is then statistically examined in order to determine whether an interaction effect is present.

Results

Four disordinal interactions were found in the analysis: "Time On The Introductory Program" predicting "Pattern Derivation"; "Time On The Introductory Program" predicting "Formula Derivation"; "Reading Aptitude" and "Errors On The Introductory Program" (together) predicting "Initial Learning"; "Reading Aptitude" and "Time On Introductory Program" (together) predicting "Formula Derivation". However, no significant interactions were found using "Reasoning Aptitude" and "Reading Aptitude" (together) as predictors for any of the outcome measures. "Errors On The Introductory Program" and "Time To Complete The Introductory Program" emerged as significant predictors—this was both unexpected and interesting; and might well be kept in mind in further research.

Conclusions

A better matching of aptitude and treatment variables seems necessary in future research, combined with treatments of longer duration. Also, it may be commented that the design and methodology used in this study has not been conventionally used by researchers. On the contrary, not uncommonly subjects are randomly assigned to treatments after aptitude measures are administered. The researchers suggest that the design and methodology used in the present study might prove to enhance prospects for uncovering significant ATI effects in future research, if coupled with treatments of longer duration.

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Thursday, April 21, 1977
3:00 - 4:00 p.m.

Panel Presentation

MATHEMATICAL READING DIFFICULTIES

Mary Ann Byrne
University of Georgia

Linda Cox
Pacific Lutheran University

Richard Lesh
Northwestern University

What kinds of resources and materials are available for mathematics teachers who must deal with reading problems, and for reading teachers who must deal with mathematical reading problems? What reading skills are involved in reading mathematics that are not commonly needed in reading other types of materials? How are problems that are involved in reading mathematics related to problems involved in understanding the underlying mathematical ideas? Who should deal with various types of mathematical reading problems?

Even though each member of our panel has recently worked on projects and conducted research on reading problems in mathematics, we do not intend to conduct a research reporting session. Research results will be mentioned when they indicate practical and useful ideas for teachers, or when they clarify important issues for teachers or future researchers; but our goal will be to clarify issues and to help make people aware of useful resources - not simply to report research. In particular, we will discuss the following types of issues and will give examples from our experiences to illustrate our points.

Results from tests, and complaints from teachers, indicate that general reading skills do not always apply to specific content areas like mathematics. But, for most reading specialists and mathematics specialists, it is not clear how reading mathematics is different from reading in other content areas. Besides the fact that mathematics uses peculiar symbols, a concise technical language, and a highly economical (e.g., non-redundant) mode of communication, some other factors also seem to distinguish mathematical reading from reading in most other content areas. Our panel will try to clarify some of these "things that people should do when they read mathematics that they may not have to do in reading most other kinds of content." We will also discuss some of the problems associated with the language and symbolism that is used in mathematics.

Even after some uniquely mathematical reading difficulties have been identified, it is not clear to most teachers who should deal with these problems. That is, which problems should be handled by a reading specialist and which problems must be handled by the mathematics teacher. The fact is that many mathematical reading problems cannot be separated

from problems involving understanding of the underlying mathematical ideas. So, our panel will try to help clarify some ways that mathematical reading problems are related to problems involving mathematical concept acquisition.

For those mathematical reading problems that should be handled by a reading specialist, some issues that arise are: (a) how can referrals be made, (b) what techniques and instruments are available for identifying children with reading difficulties, and what techniques are available for diagnosing difficulties of individual children, (c) how can mathematics teachers be sensitized to the range of reading skills that exist in most classes, and to the difficulties that can arise because of these difficulties, (d) how can reading specialists be sensitized to the fact that the skills in reading mathematics are somewhat different from reading a newspaper? For instance, it is a well known joke that if you want to make someone a poor reader, one of the best things to do is to send him to graduate school in mathematics. So, once again, it is important to point out how reading mathematics can be different from reading other types of materials.

Although national assessment data in reading seem even more discouraging than those in mathematics, reading specialists are coming up with (a) useful ways of identifying children who are having mathematical reading problems, (b) useful ways of identifying materials that will present reading difficulties, (c) techniques for identifying particular reading difficulties in mathematics, and (d) useful techniques for correcting these mathematical reading difficulties. Furthermore, some of the techniques for dealing with "mathematical reading problems" seem to be equally effective in increasing youngsters' understanding of the underlying mathematical ideas. So, our panel will review some of the materials and techniques that seem to furnish promising reading resources for mathematics teachers.

Finally, our panel will mention some pros and cons that can be involved in using "high readability" mathematics materials or materials where very little independent reading is expected from the youngster.

Thursday, April 21, 1977
4:30 - 5:30 p.m.

Research Reporting Session
Paper One

AN INVESTIGATION OF READINESS FOR THE WRITTEN SYMBOLIZATION OF ADDITION AND SUBTRACTION AT THE FIRST GRADE LEVEL

Katherine B. Hamrick
University of Georgia

William D. McKillip
University of Georgia

Purpose

The purpose of this study was to investigate the relationship among children's understanding of mathematical concepts, written symbolization of these concepts, and a well defined "readiness" for written symbolization based on verbal facility with the concepts to be symbolized. It was hypothesized that readiness for written symbolization, defined in terms of verbal facility, would influence the course of learning and the success of instruction. In order to test this general hypothesis, a teaching experiment was conducted which utilized the learning of addition and subtraction concepts and the symbols which express them for small whole numbers at the first grade level. There are a number of factors which must first be discussed in presenting the rationale of the study.

Conceptual Framework

While some of the errors children make in elementary mathematics are a result of lack of understanding of the mathematics itself, others seem to be the result of confusion associated with the written symbolization of the mathematics. It is possible that it is the written symbols that lack meaning to a child.

The meaningful learning of the written symbolization of mathematics is similar in many respects to the meaningful learning of the written symbols of language, that is, learning to read. The similarities between arithmetic symbols and language symbols and the learning of each were summarized by Hickerson (1959). In his list of similarities, Hickerson implied that oral language and oral arithmetic symbols should be learned prior to written symbols. Meaning is assigned to written symbols only when a child relates them to already known spoken symbols. Hickerson's implications are supported for language symbols by educators in reading and language education, by theories of language development, and by physiological evidence in the form of the existence of subvocal speech. Thus when one considers the similarities between language and arithmetic symbols, it is plausible that Hickerson's implications are also true for arithmetic symbols. In other words, verbal facility with arithmetic symbols is an important readiness factor learning the written arithmetic symbols.

In this study, readiness was defined as follows: Given a topic in elementary mathematics, there are lists of objectives, the attainment of which indicates mastery of the topic. Omitting those objectives that are concerned with reading, writing, or speed of response, a child is ready for the introduction of written symbolization of the topic when he has mastered the objectives of the topic verbally, perhaps with the aid of pictures or manipulatives. A specific definition of readiness was then developed based on the objectives for addition and subtraction used in the subject school.

Procedures

The subjects were 38 first grade students in Barrow School, Athens, Georgia. The subjects were classified as "ready" or "not-ready" by scores on a readiness test based on the preceding specific definition of readiness. The subjects were then paired by means of readiness test scores and Key Math Test scores. This pairing procedure resulted in eleven pairs of "not-ready" subjects and eight pairs of "ready" subjects. One member from each pair was randomly assigned to a delayed symbolization group and the other member assigned to an immediate symbolization group.

All subjects received twelve weeks of instruction on introductory addition and subtraction. The immediate symbolization groups of both "ready" and "not-ready" subjects experienced a treatment in which written symbolization was introduced simultaneously with the concepts. The delayed symbolization group of "ready" children experienced a treatment in which written symbolization was delayed for five weeks. The delayed group of "not-ready" children experienced a treatment in which written symbolization was delayed until the child had verbally mastered the objectives of the readiness test. The subjects were given a posttest designed to measure the subjects' ability to interpret given number sentences, to produce number sentences when given actions on sets, and to state answers to given number sentences.

Results

If a child is ready for the introduction of symbolization, immediate introduction of the symbolization should facilitate learning. Therefore it was hypothesized that of the children classified as "ready", those who experience immediate introduction of symbolization would be better able to interpret, produce, and state answers to number sentences than children who experience a delay of written symbolization. Preliminary data analysis, using the Wilcoxon matched-pairs-signed-ranks test, found no significant differences between the scores of the two groups of "ready" children.

If a child is "not-ready" for the introduction of symbolization, the child should learn more efficiently and the learning be more meaningful if the topic is delayed until the child masters the prerequisites in the definition of readiness. Therefore it was hypothesized that of the children classified as "not-ready", those who experience a

delay of written symbolization until they are determined to be ready would be better able to interpret, produce, and state answers to number sentences than children who experienced immediate introduction of symbolization. Examination of the data showed the scores of "not-ready" subjects in the delayed symbolization group were consistently higher than scores of "not-ready" subjects in the immediate symbolization group in all areas of the posttest. Preliminary data analysis using the Wilcoxon matched-pairs-signed-ranks-test found that the subjects in the delayed symbolization group had significantly higher ($p < .05$) scores on interpretation number sentences than subjects in the immediate symbolization group.

Conclusions

Assuming that this study is replicable and can be generalized to other topics, it may have some implications for mathematics education. The study has implications in the area of curriculum planning in that development of verbal facility with a topic should be attained prior to the introduction fo the symbolization of the topic. The study has implications for teacher training programs. Teachers would be trained to assess readiness through vocabulary analysis. The study also suggests the need for replication or parallel studies which focus on different topics in arithmetic. The results of the study also indicate that future studies concerned with meaningful learning of arithmetic should consider a child's verbal facility with the topic to be learned as a variable to be investigated or controlled.

Reference

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Thursday, April 21, 1977
4:30 - 5:30 p.m.

Research Reporting Session
Paper Two

THE RELATIONSHIP OF AREA MEASUREMENT AND LEARNING INITIAL FRACTION CONCEPTS BY CHILDREN IN GRADES THREE AND FOUR

Douglas T. Owens
The University of British Columbia

Purpose

There are several physical models which are used in school mathematics to introduce fractions concepts. One of the earliest models used is a region such as a rectangular or a circular one which has been partitioned into n congruent parts. The entire region is defined as one whole unit and each of the parts is defined as 1/n of the region. Curriculum materials often show one or more of the n parts beside the region on the printed page. It apparently is assumed that the child can measure the whole region or certain parts of it in terms of these smaller parts with measure 1/n. On the other hand, some children in grades three and four do not have well developed area concepts.

The purpose of this study was to determine the effect of the child's area concept on the ability to learn fraction concepts using area models. If a higher level area concept is helpful in learning fractions, it appears that appropriate activities in area measurement should aid fraction learning and should precede fraction activities.

A second purpose was to determine the effect of grade level on the ability of the children to learn fraction concepts at the third and fourth grade levels.

Procedures

The 56 subjects were chosen from two third grade and two fourth grade classes in Greater Vancouver, British Columbia.

The Area Concept Test was composed of six items. The test included two conservation of area items and two measurement of area items similar to those used by Piaget (1960). Both kinds of items were included because it is not clear that ability to perform one of these tasks is necessarily prerequisite to performing the other kind of task (Taloumis, 1975). In the other two items the child was asked to measure a region in terms of a set of blue rectangular cards and a set of red cards. In one item it took the same number of blue as red cards and in the second item it took fewer blue than red units. The child was then asked to measure a second region using the red cards and predict if it would take the same number or more or fewer blue cards than red cards to cover the second region. The Area Concept Test was given in a one-to-one interview and audio recorded. In each item the child was asked to justify his response.

The unit of instruction was based on a revision of the material used by Muangnapoe (1975). The instruction included indentifying fractional parts of regions using oral names, written word names and fraction symbol names. Fraction notation was used for unit fractions, other numbers less than one, equal to one and greater than one. However mixed forms were not used for numbers greater than one. Order was included for some cases where the fractions had the same denominator or same numerator, and equivalent fractions were not necessary.

The main instructional techniques were paper folding by teacher demonstration and by each child, and using appropriate oral language. Later the children completed worksheet exercises using their material kits and finally completed worksheets without the use of the materials.

The Fraction Concept Test of 51 items was of a similar nature to the worksheets completed by the pupils. The transfer test was composed of eight items sampling extension to equivalence, mixed numbers, and the set meaning of fractions.

Sequence: The Area Concept Test was administered to the pupils in grades three and four. Pupils who scored two or less were classified as low and pupils who scored four or more were classified as high. Two instructional groups were formed. Each group contained a mixture of third and fourth grade and high and low level children. The investigator instructed all groups in the same treatment for seven forty-minute periods. The posttests were administered on the eighth day with no time limit.

Analysis

The Fraction Concept and Transfer data were analyzed using separate univariate analyses of variance. The two factors, Area Concept and Grade, had two levels each. Correlations were computed among the scores on Area, Fraction Concept and Age in months.

Results and Conclusions

In the analyses both with Fraction Concept and Transfer as criteria, Area Concept was a significant factor ($p < .01$), but in neither case was Grade significant. The means for Area Measurement levels are given below:

	Low Level	High Level	Number Items
Fraction Concept	35.0	42.3	51
Transfer	1.5	3.3	8

The correlations are given in the table below. Only the correlation between Area Concept and Fraction Concept was significant ($p < .01$).

	Age	Fraction Concept
Area Concept	.05	.50
Age		.07

It was a surprising result that grade or age had no relationship to either area measurement or fraction learning. Perhaps one year's difference at this particular age is not a great enough age span to expect differences in performance. For the children in the sample at least, it appears that the children in grade three and grade four are equally capable of informal area measurement. Children in the third grade also appear to learn an initial fraction concept as well as those in the fourth grade. Again the sample was from one school and it remains to be seen how general this result is.

It does appear that area concept does have an effect on fraction learning, at least when the fraction work is based on an area model. This is not to say that the children in the low area group can't learn the fraction work. In the present study the low group mean was at about 70%. Perhaps it does mean that the high area group could learn fraction concepts more quickly or more efficiently.

While there were significant differences in the high and low area groups on the transfer measures, the means were fairly low. Perhaps in the limited time for instruction, achievement was not at a level sufficient for transfer to occur.

References

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Thursday, April 21, 1977
4:30 - 5:30 p.m.

Research Reporting Session
Paper Four

PHYSIOLOGICAL MEASURES OF CHILDREN'S MATHEMATICS PERFORMANCE: ADDITION AND MULTIPLICATION

Fredricka K. Reisman
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John T. Braggio
University of North Carolina at Asheville

Sherryll M. Braggio
University of North Carolina at Asheville

Jeff Farr
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Larry Simpson
University of North Carolina at Asheville

Purpose

To date studies of how children perform elementary school mathematics have relied upon instruments that are indirect measures of the construct being observed. Attempts at tapping pupil attitudes toward arithmetic have been in the form of attitude scales (Dutton, W. H., and Blum, M. P., 1968; Capps, L. R. and Cox, L. S., 1969) or by ranking or self report procedures. Studies stemming from the developmental stages theory of Jean Piaget have also used indirect measures where a child performs a task and inferences are made based upon observable behaviors. Decisions about sequencing and selection of mathematics curricula have relied upon techniques employing task analyses procedures based upon hierarchal theories.

The purpose of the present investigation was to examine the use of physiological measures to reflect the role of cognitively related factors in children's arithmetic performance. Also, to observe if physiological measures reflect differential effects of anxiety in mathematics performance two dependent measures were recorded, laryngea electromyogram (LEMG) and mean respiration amplitude (R).

A significant goal of this research was to devise a methodology for separating out sub-vocalization from generalized anxiety. In the present study we attempted to devise a physiological index that is thought to reflect more closely the operation of cognitive factors in children's arithmetic performance. It is suggested that the LEMG reflects the summation of sub-vocal speech and general muscle tension. The R measure may only reflect general muscle tension. By subtracting R from LEMG, using a scaled score procedure, it was possible to obtain an index of physiological activity that reflects sub-vocal speech. Sub-vocal speech may be considered a reflection or index of cognitive activity.

Conceptual Framework

Many physiological measures have been used to assess the relationship between anxiety and performance on cognitive tasks (Sayers, 1971; Zwaga, 1973). This relationship has been described as in inverse "U" function (Levitt, 1967), since some anxiety facilitates performance while a great deal of anxiety disrupts performance. Also, the amount of anxiety needed to disrupt performance decreases as task complexity increases (Lazarus, 1966). Thus far relatively few studies have extended this generalization to arithmetic tasks, save the report by Ludbrook and Vincent (1974). Therefore, this investigation attempted to determine the presence of systematic changes in the laryngeal electromyogram (LEMG) and respiration (R) during the time that children perform selected computations.

Procedures

A group of 15 children with a median age of 11 years (range 8 to 14 years) were participants.

This sample consisted of 8 males and 7 females. Response measures of LEMG and R were recorded on each participant. The LEMG was recorded by placing two silver electrodes about 2.5 cm on either side of the trachea of the participant. In this position it was possible to measure both general muscular activity due to stress and changes in muscular tension resulting from sub-vocal speech. In all cases the LEMG recordings were made using a Grass Model 7 Polygraph. The leads from the two electrodes served as input to a Grass 7P5A Amplifier. The amplified signal was fed into a Grass Driver Amplifier and the output from this unit was displayed as pen deflections on a continuous written record. In all cases R was measured by securing a strain-gauge around the diaphragm of each participant and recorded.

Each participant was tested in a sound-attenuated chamber with a one-way window. The examiner made an effort to tell the child what was to follow; i.e., placement of apparatus, presentation of arithmetic examples, and that refreshments (cookies, ice cream and candies) would be given at the end of the session. The apparatus was then positioned on the participant who was asked to sit in a relaxed position so that a baseline could be established for the two physiological measures. In most cases it was found that the overall activity of the LEMG appeared to become regular and that the R measure stabilized within several minutes. Also, spontaneous and/or elicited comments on the part of the subject suggested that the recording leads did not appear to make the participants uneasy.

At the end of the habituation period each participant presented with various computational examples among which were 5 addition and 5 multiplication items. Each child was then instructed to select any problem on the page. Steps were taken to control time allowed for cognitive interaction with each problem. The child was allowed to work on a problem for 30 sec.

Results

The Mann - Whitney *U* test was used to make independent group comparisons and the Wilcoxson Matched Pairs Signed - Ranks test for the dependent group comparisons (i.e., comparison between kind of problem, incorrect vs. correct responses, difficulty of problem). These two non-parametric tests are described by Siegel (1956); for all group comparisons alpha was .05.

Conclusions

Inspection of derived measures of LEMG appeared to reflect the relative contribution of both general physiological activity and sub-vocal speech. The relative proportion of LEMG activity of highest and lowest ranked performers on the mathematics tasks differed as a function of arithmetic operation used. For the 5 addition problems the Low Group showed less LEMG activity than that shown by the High Group. However, for the 5 multiplication problems the relationship between the Low and High Groups was reversed. These data suggest that the amount of physiological activity generated by the Low Group may not be appropriate to the task. With regards to the addition problems the Low Group did not appear to generate enough LEMG activity as compared to the High Group. On the multiplication items the Low Group appeared to generate too much LEMG activity relative to the activity of the High Group.

Inspection of the data indicates that the amount of sub-vocal speech activity appears to be less for the addition problems than for the multiplication problems. This held for the graphs of the individual participants and for the group graph.

There appeared to be a relationship between the number of problems missed and the difference in sub-vocal speech found for the two kinds of arithmetic computations. As the number of incorrect responses increased the degree of differentiation found between the two kinds of problems became less. Therefore, these data suggest that the amount and/or kind of cognitive activity demonstrated on these arithmetic problems may be a function of the arithmetic competence of the participant.

On the basis of these data it seems clear that children's anxiety on arithmetic problems can be tapped by the LEMG and R measures. The fact that the amount of physiological activity in these two measures increased as a function of correct/incorrect responding and problem difficulty suggests that anxiety plays an important role in the acquisition of arithmetic skills and concepts. As such, it is hypothesized that children of the same intellectual capacity may differ in their mastery of mathematics because of differences in their tolerance for anxiety.

Can, then, modification of LEMG by means of bio-feedback procedures affect competence in arithmetic?

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Thursday, April 21, 1977
4:30 - 5:30 p.m.

Research Reporting Session
Paper Three

THE EFFECTS OF TWO INITIAL INSTRUCTIONAL SEQUENCES ON THE LEARNING OF THE CONVENTIONAL TWO-DIGIT MULTIPLICATION ALGORITHM IN FOURTH GRADE

Donald W. Hazekamp
Central Michigan University

Purpose

In recent years there has been concern about decreasing computational proficiency. Whether this decline is due to improper development of computational procedures or the lessening of commitment to computation as a goal is not known. But, there is evidence of the lack of studies on this topic in the last twenty years.

This study was designed to investigate the relative effectiveness of two initial instructional sequences on: 1) the learning of background ideas and techniques basic to the learning of the two-digit multiplication algorithm, and 2) the learning of the two-digit multiplication algorithm itself.

Each of the initial sequences presented a different meaningful approach to the teaching of the prescribed mathematical content. One initial sequence, Treatment A, highlighted the use of grouping and base ideas. The properties of the whole numbers were used implicitly together with informal reasoning patterns in the development of concepts. The second sequence, Treatment B, focused on place value representations. The properties of whole numbers, particularly the commutative and associative properties of multiplication were used explicitly and in many of the lessons formal reasoning patterns were used in the development of concepts.

In both sequences models and symbolism reflecting their particular emphases were used. In Treatment A, the redevelopment of renaming concepts and the structure of the decimal numeration system was done using base ten blocks, hundred squares, or ten strips and the use of word-name numerals. In Treatment B this redevelopment was done using place value models as an abacus or tally charts with numerals expressed in expanded notation.

The multiplying of a 2-digit number by a multiple of ten was done as follows:

In Treatment A	In Treatment B
By establishing patterns as:	By use of the properties of multiplication develop the concept by working out example
1 x 23 = 23 1 ten x 23 = 23 tens so 10 x 23 = 230	as: 20 x 23 = (2 x 10) x 23 = (10 x 2) x 23 = 10 x (2 x 23) = 10 x 46 = 460
2 x 23 = 46 2 tens x 23 = 46 tens so 20 x 23 = 460	
3 x 23 = 69 3 tens x 23 = 69 30 x 23 = 690	or 23 x 30 = 23 x (3 x 10) = (23 x 3) x 10 = 69 x 10 = 690

In Phase II, after the initial sequences, all pupils were taught the short vertical computational form for finding the product of two 2-digit numbers in which only two partial products are computed.

Example:

$$\begin{array}{r} 38 \\ \times\ 24 \\ \hline 152 \\ +\ 760 \\ \hline 912 \end{array}$$

Procedures

The study was divided into three phases:

Phase I (8 days): Initial sequences taught

Phase II (4 days): Identical lessons on conventional two-digit multiplication algorithm taught to both groups

Phase III (12 days): Retention period

Ten regular fourth grade teachers from Mt. Pleasant and Clare, Michigan, were randomly assigned and taught the author-developed lessons. Data obtained from the author-developed tests were analyzed by analysis of covariance using the Metropolitan Mathematics Achievement Test and a screening pretest as multiple covariates.

The two treatment groups were divided into high, middle, and low achievement level subgroups. Analysis of variance was used for comparing

the achievement level subgroups. A .05 level of significance was used for testing all hypotheses. Item analysis and error analysis were also performed.

Measures were obtained for understanding of concepts, computation, transfer, multiplication thinking strategies and attitudes.

Results

1. Group A performed significantly better than Group B on concepts and computational skills taught in Phase I. The two groups scored approximately the same on concepts but Group A was significantly superior on the nine computational items.

An examination of the errors on the computational items revelaed that Group B made many more place value errors.

2. There was a significant difference favoring Group A on comprehension and computation with the conventional two-digit multiplication algorithm. Consistent with errors made on the Phase I Test, B pupils continued to have difficulty multiplying a two-digit number by a multiple of ten.

Both A_L and A_M achievement subgroups had means that were significantly higher than their corresponding B_L and B_M subgroups on the computational items.

3. On each of the three computational transfer tests Group A had the greater means. Significant differences favoring the A group were found on the tests given at the end of Phase I and Phase II.

4. No significant differences were found on the two multiplication-estimation thinking strategies tests. Group A had greater means than Group B on both measures, but the performance of both groups was below fifty percent.

5. No significant difference was found between the two groups on the computational retention test.

6. For achievement subgroups, no significant differences were found between A_H and B_H. The middle achievement group comparisons showed significant differences favoring A_M on the computational and computational transfer tests given at the end of the first two phases.

The means of A_L were greater than B_L on all tests and subtests. Significant differences were found on four of the five computational tests. On the initial mathematics achievement test, B_L scored significantly higher than A_L.

7. No significant differences were found between changes in attitude toward school. Significant differences in changes of attitude toward mathematics and toward multiplication were found. The attitudes of Group A remained relatively stable and the attitudes of Group B declined.

8. Less than one-fifth of the errors involved multiplication facts. The majority of the errors were in the second partial product and consisted primarily of multiplication regrouping and place value errors.

Conclusions

The major conclusion was that the initial A Treatment sequence resulted in better computational performance with the two-digit multiplication algorithm. In particular, it provided the better background for the low achievement level pupils. The results lend support to the contention that base representations are helpful in algorithmic work, and suggest that an informal approach using base representatives may be more appropriate for the initial learning of the conventional multiplication algorithm than a more formal approach using place value ideas and properties.

Friday, April 22, 1977
9:00 - 10:00 a.m.

Round Table Discussion

A STUDY OF THE EFFECTS OF TWO STRATEGIES FOR TEACHING TWO MATHEMATICAL SKILLS

Thomas J. Cooney
University of Georgia

Edward J. Davis
University of Georgia

James J. Hirstein
University of Georgia

Purpose

In recent years there has been a concern over the acquisition of basic mathematical skills by students. Some have coined the phrase "back to basics" to describe this concern. One of the primary issues of the back to basic emphasis is the relative importance of understanding a mathematical skill versus performance of that skill.

The present study attempted to investigate the learning of mathematical skills with particular emphasis on the role of understanding and practice in the acquisition of the skills. The specific question raised was whether the treatment of practice followed by understanding was more effective or less effective than a treatment of understanding followed by practice. After a pilot study in which this basic question was considered the following investigation was designed.

Procedures

There were two treatments--practice-understanding (P-U) and understanding-practice (U-P)--devised to teach two skills; (1) a Tractenberg method for multiplying any number by a two digit number, and (2) finding the product of two binomials. The teaching strategies devised for P-U and U-P were described in terms of moves for teaching mathematical skills as these moves are defined by Cooney, Davis, and Henderson (1975). In general, practice moves were concerned with helping students understand how to perform the algorithms. These included demonstrations of the algorithms and practice exercises in performing the algorithms. Understanding moves concentrated on explaining why the algorithms work.

The study investigated the effects of the two treatments on learning each skill. Treatment 1 (P-U) for each of the skills is presented below.

	Skill 1	Skill 2
Day 1	Introduction & Practice moves for multiplying by 12	Introduction and practice moves for multiplying binomials like $(\square + 2)$. $(\square + 4)$.

	Skill 1	Skill 2
Day 2	Practice moves for multiplying by 26 and 54.	Practice moves for multiplying any two binomials.
Day 3	Understanding moves for why the procedure works with 12.	Understanding moves for why the procedure works for $(\square + 2) \cdot (\square + 4)$ and others.
Day 4	Understanding moves for why the procedure works with 26 and 54.	Understanding moves for why the procedure works for multiplying and two binomials.

Treatment 2 (U-P) for both skills was also 4 days in length. The first two days consisted of the same understanding moves as described in days 3 and 4 above. The third and fourth days concentrated on practice moves as described in days 1 and 2 above.

Treatment 1 for skill 1 was taught by one investigator and Treatment 2 was taught by a second investigator for the same skill from January 19-22, 1976. A posttest was given on January 23 and the retention test on February 18. The second skill was taught from February 23-26 with the first investigator teaching skill 2 using Treatment 2 and the second investigator teaching skill 2 using Treatment 1. A posttest was given on February 27 and a retention test for skill 2 followed on March 24. Each of the daily 45 minute teaching sessions was audio-recorded to insure the fidelity of the treatments or to provide information in the way that the sessions deviated from the planned treatments.

Evaluation Instruments

Both posttests and both retention tests were administered individually by trained testers. Each session was audio-recorded. The tests were designed to measure three aspects of skill acquisition: speed and accuracy in performing the skill, understanding the skill, and an extension of the skill to a new but related problem. Speed was determined by timing how long it took a subject to complete given exercises. Understanding items were concerned with ascertaining the subjects understanding of how basic principles, e.g., the distributive law, related to the skills. Extension of the first skill required the subjects to apply the Tractenberg method to multiplying by a three digit number. The extension item for the second skill involved multiplying a binomial by a trinomial.

Selection of Subjects

A pretest was administered to two sixth grade classes of 25 students most of whom were consistent B students. The pretest consisted of performing basic calculations, demonstrating and understanding of the associative law for multiplication and the distributive law, and using variables. Ten matched pairs were selected on the basis of IQ and pretest

scores. Constraints placed by the participating school required that each matched pair contain one student from each class. The IQ range for the subjects was 110-125.

Analysis

A comparison of the means of the two treatment groups will be made for speed and accuracy. The null hypotheses that the means are the same for the two groups can be tested using a t-test for matched pairs. A comparison of the two groups on understanding and extension variables will be based on the quality of subjects' responses to those items.

Friday, April 22, 1977
9:00 - 10:00 a.m.

Panel Presentation

A CONSORTIUM OF RESEARCH PROJECTS CONCERNING SPATIAL AND GEOMETRIC CONCEPTS

A. I. Weinzweig
University of Illinois, Chicago Circle Campus

Alan Hoffer
University of Oregon

Arthur Coxford
University of Michigan

Larry Martin
Missouri Southern State College

In the Spring of 1975, a research workshop was held as part of the activities of the University of Georgia Center for Learning and Teaching Mathematics. The workshop was attended by more than thirty participants from throughout the country, and papers were presented by Izaak Wirszup from the University of Chicago, Jacques Montangero from the International Centre for Genetic Epistemology, Geneva, Switzerland, Richard Lesh from Northwestern University, Charles Smock from the University of Georgia, Edith Robinson from the University of Georgia, and Michael Mitchelmore from the Ministry of Education, Kingston, Jamaica. These papers and proceedings of this conference have been published in a monograph titled Research Concerning the Development of Spatial and Geometry Concepts (Eric Information Center, 1976).

Following the Georgia Workshop, participants planned a coordinated series of research projects to be conducted throughout the 1975-76 academic year. The projects were based on papers and discussions that took place at the Georgia Workshop. The following working groups were formed:

1. Isometric Transformations (i.e., Rigid Motions)

   Diane Thomas - Ohio State University
   Faustine Perham - University of Illinois Chicago Circle Campus
   Richard Kidder - Longwood College, Farmville, Virginia
   Karen Schultz - Georgia State University
   Judy Musick - Northwestern University

2. Non-Isometric Transformations (i.e., Similarity, Topological Transformations, etc.)

   Arthur Coxford - University of Michigan
   Larry Martin - Missouri Southern State College
   Karen Fuson - Northwestern University

3. Perception and Imagery

Richard Lesh - Northwestern University
Naomi Fisher - Northwestern University
Don Balka - Saint Mary's College, Notre Dame, Indiana
Greg Bauer - Indiana-Purdue, South Bend, Indiana
Cheryl Gay - Northwestern University

4. Coordinate Systems

Howard Johnson - Syracuse University
John Moyer - Marquette University
Bill Juraschek - University of Colorado
Roger Chilewski - St. Xavier College, Chicago, Illinois
Ken James - Roosevelt University, Chicago
Paul Trafton - National College of Eduation
Hazel Wagner - Chicago Board of Education Research Department
David Bradbard - University of Georgia

5. Von Hiele Levels

David Cilley - IBM, New Jersey
Alan Hoffer - University of Oregon
A. I. Weinzweig - University of Illinois, Chicago Circle Campus

6. Cross Cultural Studies

Jeffrey Barnett - Northern Illinois University
Charles Dietz - Northern Illinois University
Theodore Eisenberg - Northern Michigan University

A second research workshop was held at Northwestern University during the Winter quarter of 1975-76. The purpose of this meeting was to coordinate the research reports from the studies that were being completed during the 1975-76 academic year, and to make plans for a monograph to publish this research as a unified set of studies. The papers for this monograph are currently in final draft stages, and the monograph is scheduled to appear in September, 1977 from the Eric Information Center.

A third meeting of the space and geometry working group was held at the 1976 NCTM meeting in Atlanta, and a fourth meeting is planned to be held at Northwestern University in September, 1976. The purpose of these meetings was to continue to coordinate and synthesize past research efforts and to plan for future research projects.

Although our "space and geometry" research group has completed a number of related projects during the past year, and although a series of coordinated studies have already been planned for next year, we do not simply want to report the results of our studies. There have simply been too many projects completed. Instead, we would like to briefly summarize the major conclusions that can be formed by looking at all of our research efforts taken as a whole. We would also like to discuss the major unifying themes that run through all of the individual studies we

have completed, and to outline some key questions that we believe should be the target of future research efforts. A primary focus will be to describe how our research work in space and geometry relates to research in other areas of concept acquisition (i.e., number and measurement ideas, problem solving, etc.), and to point out practical implications for teacher trainers and textbook authors.

We believe that the only way we can find answers to most of the important instructional problems that teachers face is if groups of people work together cooperatively over a long period of time on coordinated research efforts. So, we will try to make it easy and attractive for other individuals to participate in the activities of our group.

Friday, April 22, 1977
10:30 - 11:30 a.m.

Round Table Discussion

DIFFERENCES BETWEEN TEACHERS' SELF-RATINGS AND PRINCIPAL AND UNIVERSITY FACULTY'S IDEALIZED MATHEMATICS TEACHER AS MEASURED BY A MATHEMATICS INVENTORY

Thomas R. Post
University of Minnesota

William H. Ward, Jr.
University of Minnesota

Victor L. Willson
University of Minnesota

Purpose

This survey was conducted to examine differences and similarities between teachers', principals', and college professors' views as reflected on an inventory concerned with both cognitive and affective outcomes of mathematics instruction. Scores were compared using multivariate analysis of variance procedures.

Procedures

The Minnesota Research and Evaluation Project, was funded to study the process of educational change and evaluate a number of NSF supported projects in selected regions of the country. As part of its total effort, in 1972, 222 secondary level principals in California, Michigan, and Indiana were randomly selected to participate in a data gathering procedure. Each principal was asked to randomly select one teacher from the mathematics faculty of their school. Both teachers and principals subsequently completed a battery of instruments, among these the 30-item Mathematics Inventory for Teachers (MIT). The MIT assembled by Bracht (1972) contains statements concerning educational ideas and practices specifically dealing with the teaching and learning of mathematics in the secondary school. The items assess attitudes toward mathematics and its teaching. Teachers were asked to express their own beliefs and opinions, principals were asked to respond to the MIT as they <u>believed</u> an <u>ideal</u> mathematics teacher would.

University level participants in this study were selected from the mailing list of the Bulletin for Leaders from the National Council of Teachers of Mathematics. Forty-eight states (Alaska and Hawaii excluded) were stratified into six geographic regions. Participants' names were randomly selected from each region. A total of two hundred names were generated using this procedure. One hundred forty-one responded.

University level participants were asked to respond to the MIT "as you would expect an ideal mathematics teacher to answer the items," the same directions as were given principals.

Conceptual Framework

In a precursor to the present study, Post, Ward and Willson (1974) used factor analytic techniques to sort the 30 MIT items into eight factors and calculate factor scores for both principal and teacher in 148 randomly selected cases (principal-teacher pairs) (P-T). The vectors of principal-teacher factor score differences (P-T's) on seven identifiable factors* were tested via multivariate analysis of variance procedures against the hypothesis of no differences between a principal and his teacher ($H : \overrightarrow{P-T}=\vec{0}$). The principal-teacher difference on two factors, "Teacher Concern for Student" and "Higher Order Concern" showed different from 0 at $P<.01$ and .07 respectively. Principal-teacher differences were not detected on the other factors. The weightings used in determining factor scores were derived from the factor analysis of teacher's responses only. A totally independent factoring of principal's responses, however, yielded an essentially identical factor structure. This noteworthy stability would indicate that principals and teachers conceptualize the issue implied by the MIT in fundamentally the same manner.

Analysis

It was originally anticipated that an analysis parallel to the above would be performed in the present study. However, when factor analysis was performed on the MIT item data generated by college and university mathematics educators, difficulties arose. The resulting factor structure was not only seemingly unidentifiable, it did not at all correspond to the structure generated in the earlier study referred to above. Such differences imply that college and university mathematics educators do not conceptualize mathematics education issues (at least those identified as factors in the earlier study) in the same manner as school personnel. Such differences raise the possibility that these groups are fundamentally different, each ascribing to ideas, procedures, and beliefs which are not only diverse but, also, perhaps misunderstood to one another. If substantiated by further research these differences would inhibit the rapidity with which educational innovations are implemented in the school setting and would further help to explain why many of the types of methodological and ideological changes normally suggested by university level mathematics educators are not realized at the classroom level.

The question was raised as to whether university level elementary and, university level secondary mathematics course instructors differed in their responses to the MIT in the multivariate sense. If so, specific areas of discrepancy would be pinpointed using univariate F procedures. Two such analyses were performed. (1) Contrasting those persons who

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*These factors were labeled: (1) Flexibility, (2) Mathematics as Process, (3) Teacher Concern for Student, (4) Vocational Satisfaction, (5) Non-Rigid Practices, (6) Attitude Toward Teaching, and (7) Higher Order Concerns. (Factor 8 was not discernible.)

identified themselves as elementary methods only and secondary methods only, and (2) Contrasting those persons who identified themselves as solely elementary methods instructors and those who indicated responsibility for both elementary and secondary methods courses. In each case, a 30-dependent-variable multivariate analysis of variance was conducted. Thirty univariate analyses were concurrently performed to suggest which MIT items contributed most to such multivariate differences as might be found.

An analysis (1), (Elementary methods only, contrasted with secondary methods only) the F-ratio for the multivarite test of equality of mean vectors (30 dependent variables = individual MIT items) = 1.2028, df = (30, 35), $p < .30$.

In analysis (2) (Elementary methods only, contrasted with elementary and secondary methods) the F-ratio for the multivariate test of equality of mean vectors (30 dependent variables = individual MIT items) = 1.1190, df = (30, 56), $p < .35$.

It was concluded from these two multivariate analyses that both elementary and secondary methods persons responded to the MIT in essentially the same manner. It is therefore reasonable to combine data from elementary and secondary level methods persons in subsequent contrasts with principals and/or teachers to increase the power of the analysis.

In contemplating the contrasts of university educators to teachers to principals, a potentially major design problem arose. The university educators were obviously an independent group; however, it was equally obvious that the teachers and principals should be treated as matched pairs. The problem could be circumvented if the principals and teachers could be treated as if they were independent samples, a treatment justified if the groups could be shown to be statistically independent.

It was concluded that such statistical dependence as did exist was slight and that treating the principals and teachers as independent groups was justified. The price of such a decision is to make certain parts of the following analysis tend toward the conservative.

When university persons were subsequently contrasted with teachers, significant differences occurred on 18 of 30 MIT Items (at $p < .05$). When principals and teachers were contrasted, significant differences occurred on only 10 of 30 MIT Items. These data imply that principals and teachers tend to be more closely aligned and express a greater amount of ideological agreement than teachers and university mathematics educators, at least with respect to their patterns of responses to the MIT Items.

The relatively large scale discrepancies obtained suggest inherent or learned differences between groups. The magnitude of differences decreases as one proceeds from overall group comparisons to university instructor-teacher comparisons, to principal-teacher comparisons.

Conclusions

Teachers seem to be more ideologically compatible with their respective principals than with university level mathematics methods instructors. This is especially noteworthy since the MIT Scale was concerned exclusively with the teaching and learning of mathematics, and it is unlikely that a large portion of principals have been exposed recently to contemporary ideas regarding the teaching and learning of mathematics, at least as those ideas are espoused by university level mathematics educators. Why would mathematics teachers be less inclined to relate ideologically to those ideas espoused by "experts" in the field of mathematics teaching than to those of their school principal whose general lack of expertise in the field is both known and accepted? At least three possible non-competing explanations can be discussed.

1) By virtue of working within a larger organization, in this case the school, constraints are placed on an individual teacher's decision making power. This phenomenon tends to result in an apparent within-school conformity of thought and idea. 2) This study implies that contemporary programs in mathematics education at the college and university level have a decidedly cognitive bend. (This conclusion comes from data not discussed in this abstract.) In possible contrast are currently used objective based mathematics programs which appear to be more behaviorally oriented. It appears that the degree of transfer from formal professional education to actual classroom implementation may thus be minimal. Whether the ideas and practices espoused in formal mathematics courses are considered simply idealistic, non-functional, or impractical is not clear at this time, but what does seem to be evident is the existence of fundamental differences in the conceptual framework within which teachers, principals and university level mathematics educators approach the teaching and learning of mathematics. 3) Since the popularity of cognitive oriented mathematics learning has occurred primarily within the last decade or so, it is conceivable that a large number of secondary mathematics teachers who have completed their formal professional education sequence prior to that time have chosen not to formally update their professional skills. Such persons having been trained in the behavioral or neo-behavioral approaches may not be at all familiar with the cognitive position, or after limited exposure may have rejected such an orientation.

We think university and college educators continually assume that the message (plea for change to a more cognitive type orientation) is eminently worthwhile and therefore acceptable at least at the intellectual level by the classroom teacher. The authors realize that at this point such an assumption is not fully substantiated and that more detailed and relevant information must in the future be accumulated. Given such an assumption, however, and considering that chnage appears slow to occur, educators at the university and classroom levels often pinpoint other factors which are responsible for "lack of progress." Frequent examples being: parents, administrators, tradition, standardized tests, school organizational patterns and school policies to mention a few. Given that teachers are more compatible ideologically with their principals than with college and university level mathematics educators, the present study suggests that secondary school teachers may be more content with the existing

behaviorial orientations in the mathematics classroom than had been heretofore expected.

This study is viewed as both preliminary and exploratory in nature. As noted earlier, several of the statements made are conjectural at this point in time. It should be noted, however, that these conjectures are not inconsistent with the data and are further supported by the author's professional experiences.

Additional research is needed to shed more light on these important issues. New and refined instruments need to be developed, written participant responses should be coupled with structured interviews and various other populations and subject diciplines might also be examined. If large scale population differences persist and further evidence is found to be consistent with the data, and speculation herein, a significant factor contributing to the indolent pace with which innovation occurs in the educational system will have been identified. Such positive identification will undoubtedly prove useful in future attempts to develop a more systematic and comprehensive approach to the improvement of curricula and pedogogy in the nation's schools.

Friday, April 22, 1977
10:30 - 11:30 a.m.

Research Reporting Session
Paper One

A STUDY OF THE RELATIONSHIPS BETWEEN SELECTED NONCOGNITIVE FACTORS AND THE PROBLEM SOLVING PERFORMANCE OF FOURTH GRADE CHILDREN

Donald R. Whitaker
University of Wisconsin-Madison

Purpose

The purpose of this study was to investigate the relationships between selected noncognitive factors and the problem solving performance of fourth grade children. Those factors investigated were children's attitudes toward mathematical problem solving, teachers' attitudes toward mathematical problem solving, and related sex and program-type differences.

Conceptual Framework

The development of students' ability to solve problems is one of the primary goals of elementary mathematics instruction, and educators continue to seek information about the nature of this ability. Favorable student attitudes toward problem solving are believed to be a desirable educational outcome (Brownell, 1942), and favorable teacher attitudes are important in helping students acquire problem solving proficiency (Polya, 1965). Aiken (1970) has called for more intensive investigations into the nature of attitudes toward mathematics and notes that an individual's attitude toward one aspect of the discipline, such as problem solving, may be entirely different from his attitude toward another phase of the discipline, such as computation. Researchers, however, have tended to use single, global measures of attitude toward mathematics, rather than investigating attitude toward only one phase of the discipline. Lindgren, et al. (1964) did use an adaptation of a measure of problem solving attitude developed by Carey (1958), but correlated the results only with arithmetic achievement and not with problem solving performance. Thus, this study evolved from a need to examine, in more detail, the relationships between measures of student and teacher mathematical problem solving attitudes and measures of student performance in mathematical problem solving.

Procedures

The mathematical problem solving test used in the study is a 22-item test which provides a measure of comprehension, application, and problem solving for each item (Wearne, in preparation). The student mathematical problem solving attitude scale is a 36-item Likert-type scale, and the teacher mathematical problem solving attitude scale is a 40-item scale with Likert-type format (Whitaker, in preparation).

The study was conducted in two parts, as depicted in Figure 1.

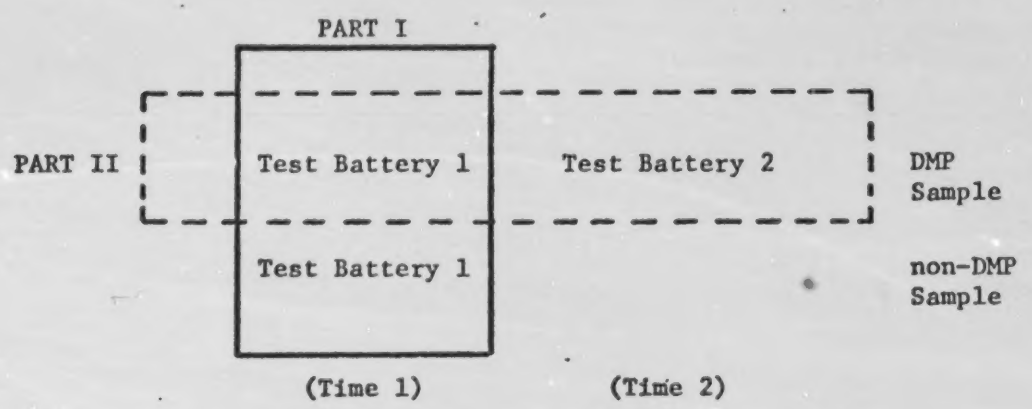

Figure 1. DESIGN OF THE STUDY

During the fourth month (Time 1) of the 1975-76 school year 30 fourth grade classes in 13 Wisconsin schools participated in the first testing. Test Battery 1 consisted of the student problem solving test, the student attitude scale, and the teacher attitude scale. Fifteen of the classes in the study were using Developing Mathematical Processes (DMP), a research-based, K-6 elementary mathematics program recently developed at the University of Wisconsin. The remaining 15 classes were not using the DMP materials. During the seventh month (Time 2) of the 1975-76 school year the 15 DMP classes participated in a second round of testing. Test Battery 2 consisted of an alternate version of the student problem solving test and the student and teacher attitudinal scales.

Analysis

Seven main questions and 11 ancillary questions are investigated. The questions for Part I deal with the favorableness or unfavorableness of student and teacher attitudes toward mathematical problem solving, the performance of students on the mathematical problem solving test, and the relationships between student performance, student attitudes, and teacher attitudes. The ancillary questions deal with sex differences and program-type differences (DMP versus non-DMP) in the above relationships.

Part II of the study is based on the cross-lagged panel correlational technique (Campbell and Stanley, 1963) as advocated by Aiken (1969). This part of the study attempts to determine the direction of effect between teacher problem solving attitudes and student problem solving attitudes and performance. Using cross-lagged panel correlations, teacher attitudes at Time 1 and Time 2 are correlated with student attitudes and performance at Time 1 and Time 2 in order to suggest which variable has the greater influence on the other.

Results

Although data analyses are incomplete at the time of writing of this proposal, preliminary findings suggest that fourth grade students and teachers have favorable attitudes toward mathematical problem solving, independent of the type of mathematics program studied. The data also suggest that DMP students perform better than non-DMP students on the measures of comprehension, application, and problem solving included in the problem solving test, although the significance of these differences is yet to be determined. A more detailed reporting of results will be possible at the time of the presentation.

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Friday, April 22, 1977
10:30 - 11:30 a.m.

Research Reporting Session
Paper Two

DEVELOPMENT OF A TEST OF MATHEMATICAL PROBLEM SOLVING WHICH YIELDS THREE SCORES: COMPREHENSION, APPLICATION, AND PROBLEM SOLVING.

Diana Catherine Wearne
University of Wisconsin-Madison

Purpose

The purpose of this study was to develop a test of problem solving which provides a measure of a child's comprehension of the information presented in the problem and his mastery of the concepts needed to solve the problem as well as a measure of problem solving behavior.

Rationale

There is unanimity among mathematics educators of the importance of developing problem solving abilities of children. The Cambridge Conference on School Mathematics (1963) indicated that problem material should receive at least half of the time and attention of authors of text series. Other groups, including the College Entrance Examination Board (1959), the National Advisory Committee on Mathematical Education (1975) and the K-13 Report Committee of the Ontario Institute for Studies in Education (1970) have all stressed the importance of problem solving activities. Thus, it would appear that a measure of problem solving is needed in order to determine to what extent this goal is being attained.

Individually administered tests of problem solving have the advantage of not only producing a score but an opportunity for the investigator to observe the child solving the problem and to ask the child how he solved the problem or, if he was unable to solve the problem, to be aware of his path of reasoning and the aspects of the problem which were confusing to him.

However, the constraint of a limited amount of time usually allotted to assess the behavior necessitates the use of a group administered test. A major difficulty associated with a group administered test is the inability of the examiner to identify the subjects who truly cannot solve the problem as opposed to those who were unable to solve the problem because they either did not understand the information presented in the question or because they had not mastered the prerequisite concepts of the problem solving question. Therefore, a test which assesses comprehension of information and relevant concepts not only produces more information about the child but also provides a "truer" measure of his problem solving ability by making it possible to consider only those problems for which the child has demonstrated preparedness.

The test was designed to yield three scores: a comprehension score, an application score, and a problem solving score. To accomplish this, the test was composed of groups of items, each group being called a superitem. Each of the superitems contained a comprehension question, an application question, and a problem solving question.

The comprehension question assesses the child's understanding of the information contained either implicitly or explicitly in the item stem. The application question assesses the child's mastery of a prerequisite concept or skill of the problem solving question. The third question in the set is the problem solving question. A problem situation was defined to be a situation which posed a question whose solution was not immediately available, i.e., a situation which did not lend itself to an immediate application of some rule or algorithm.

Procedures

The population for the study was approximately 600 fourth grade children in Wisconsin; nine schools in seven towns or cities participated.

A test composed of the type of items described (superitems) produces more information than one containing only problem solving items; however, structuring the test in this manner raises some questions.

1. Does asking a series of questions have a facilitating or debilitating effect on the response to the questions; in particular, does the inclusion of a comprehension and an application question have an effect on the response to the problem solving question?

To answer this question, six tests composed of subsets of items were administered in addition to the full test. Three of the tests contained only one type of question (comprehension, application, or problem solving) and three other tests contained two types of questions (comprehension and application, comprehension and problem solving, or application and problem solving). The chart below describes the test's content.

	Question Type		
Test	Comprehension Questions	Application Questions	Problem Solving Questions
C-test	x		
A-test		x	
P-test			x
CA-test	x	x	
CP-test	x		x
AP-test		x	x
CAP-test	x	x	x

Figure 1. DESCRIPTION OF TESTS

Means and variances of each of the three scales (comprehension, application, and problem solving) were compared across all instruments containing the scale to determine if a significant difference existed in the measures.

2. How was an estimate of reliability obtained?

Virtually nothing has been written on estimating the reliability of a test composed of superitems. A generalized KR-20 was obtained using the superitems as a unit, a procedure recommended by Cureton (1965). In addition, a Hoyt reliability estimate was found (under the assumption that if the means of the scales on the different tests containing them did not differ significantly, the items were essentially independent).

3. How was an estimate of validity obtained?

Due to a lack of a universally accepted definition of problem solving behavior and, therefore, no established criteria against which the tasks may be validated, the only indicant of validity that could be obtained is content validity. A panel of judges was given a series of item stems and questions and a definition of each of the three categories of items and asked to classify the questions. A measure of consistency of the classification of the questions was found.

4. To what extent is the model for the test supported by the results?

It is hypothesized that the child must be able to respond correctly to the comprehension question before he is able to respond correctly to the application question and must respond correctly to both comprehension and application questions before he is able to respond correctly to the problem solving question. To answer this question, the following conditional probabilities were found for each of the superitems:

$$\text{Prob}\left[\text{responded correctly to the comprehension question} \mid \text{responded correctly to the application question}\right]$$

$$\text{Prob}\left[\text{responded correctly to the application question} \mid \text{responded correctly to the problem solving question}\right]$$

$$\text{Prob}\left[\text{responded correctly to the comprehension question} \mid \text{responded correctly to the problem solving question}\right]$$

$$\text{Prob}\left[\text{responded correctly to the comprehension and application question} \mid \text{responded correctly to the problem solving question}\right]$$

Analyses and Results

The data analysis is not complete, however, some preliminary results may be reported. Projected analysis of the data is described in the following paragraphs.

T-tests were used to determine if significant differences existed between the means of the various scales (Comprehension, Application, and Problems Solving) on the different instruments containing them. This procedure, multiple t-tests, was used due to a concern for Type II errors. Two of the eighteen t-tests were deemed to be significant.

An item analysis was performed on each of the seven tests in an effort to identify items with stable item parameters.

A cluster analysis was performed and additional contingency tables corresponding to the clusters were also found in an effort to determine if the probabilities associated with the superitems remained the same for each cluster.

The data from the original contingency tables and those associated with the clusters would appear to support the underlying test model, i.e., a correct response to the comprehension question is a prerequisite to the application question, etc.

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Friday, April 22, 1977
10:30 - 11:30 a.m.

Research Reporting Session
Paper Three

A STUDY OF THE RELATIONSHIP OF MATHEMATICAL PROBLEM SOLVING PERFORMANCE AND INTELLECTUAL ABILITIES OF FOURTH-GRADE BOYS AND GIRLS

Ruth Ann Meyer
University of Wisconsin-Madison

Purpose

The primary objective of a study of Meyer (1976) was to contribute to a better understanding of intellectual abilities related to mathematical problem solving. More specifically, this study examined relationships between a cognitive ability structure identified for the population sample and the performance of the sample on a mathematical problem solving test. All analyses for this initial study were performed for boys and girls combined. This present study contains the analyses for the original population sample for boys and girls separately.

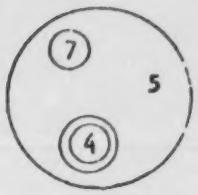
All data analysis procedures were modeled after those of A Structure of Concept Attainment Abilities Project (CAA) by Harris and Harris (1973). The CAA Project was a study of intellectual abilities and concept learning by the Research and Development Center at the University of Wisconsin-Madison.

Procedures

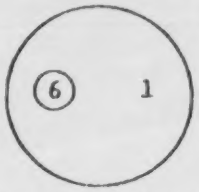
A battery of 20 tests were administered to 97 fourth-grade boys and 82 fourth-grade girls who were studying Developing Mathematical Processes (DMP), a K-6 elementary mathematics curriculum (Romberg, Harvey, Moser, Montgomery, & Dana, 1974; Romberg, 1976). Eighteen of these 20 tests were selected from the "reference" tests for cognitive abilities used by the CAA Project. The other two, a mathematics computation test (Romberg, 1975) and a mathematical problem solving test (Romberg & Wearner, 1975), were designed to assess the performance of children who are studying DMP.

The mathematical problem solving test was designed to yield three scores: a comprehension score, an application score, and a problem solving score. Each of the 19 superitems of the test contains a comprehension, an application, and a problem solving question. The comprehension question (Problem Solving 1) assesses a child's understanding of the information presented explicitly or implicitly in the item stem. The application question (Problem Solving 2) involves a fairly straight-forward application of some rule of concept to a situation. The problem solving question (Problem Solving 3) presents a situation which involves other than a routine application of some principle. Since the test differs from other mathematical problem solving tests in that each item yields three scores, an example is presented to illustrate the nature of the comprehension, application, and problem solving parts.

Item Stem

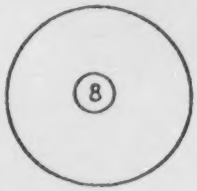
In Circleland, people write 

when they mean 475

and they write 

when they mean 61.

Problem Solving 1 (Comprehension)

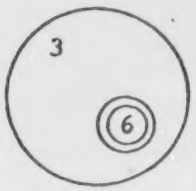
In Circleland, people write 

when they mean 8.

TRUE FALSE

Problem Solving 1 assesses a child's understanding of the information presented explicitly in the item stem.

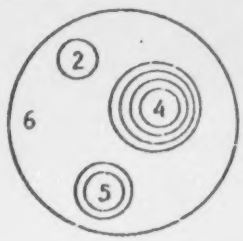
Problem Solving 2 (Application)

What do they mean when they write 

36
63
630
603
306

Problem Solving 2 can be answered by direct application of information contained in the item stem.

Problem Solving 3

What do they mean when they write 

4,526
40,526
4,562
45,620
45,260

Problem Solving 3 requires a generalization of information presented in the item stem.

Analysis

For this present study, total performances on Problem Solving 1, Problem Solving 2, and Problem Solving 3 were analyzed separately.

After scoring all tests, means, standard deviations, and Hoyt analysis of variance reliability estimates were obtained. In general the reliability estimates were quite good, with only four estimates below .70 for girls and five below .70 for boys. The mean scores and standard deviations were similar for boys and girls. T-tests demonstrated significant differences for only Spatial Relations and Picture Group Name Selection.

Single battery factor analysis and canonical correlation suggested differences in the mathematical problem solving structures of boys and girls. When the 18 reference tests for cognitive abilities, the mathematics computation test, and the three problem solving scores were factor analyzed, five factors emerged for boys and six factors emerged for girls. The factors for boys included: (1) Verbal ability and Word Fluency, (2) Induction of classes employing symbolic or figural content, (3) Perceptual Speed, (4) Problem Solving, and (5) Mathematics Concepts. The six factors identified for girls were: (1) Verbal ability; (2) Induction of classes employing pictorial, figural, or verbal content; (3) Numerical ability; (4) Perceptual Speed; (5) a Fluency factor employing either words or numbers; and (6) General Mathematics.

Two significant canonical variates were found for boys. The first canonical variate was identified as a Concept Attainment dimension and the second variate appeared to be a Problem Solving dimension. The only significant canonical variate for girls was Concept Attainment with a problem solving component.

Significantly, two of the five factors identified for boys by factor analysis were mathematics factors. One of these, the Problem Solving factor, was determined primarily by Problem Solving 3. Whereas the other mathematics factor, Mathematics Concepts, was determined by Problem Solving 1 and Problem Solving 2. Only one mathematics factor was identified for girls. This factor, General Mathematics, was determined by Problem Solving 1, Problem Solving 2, Problem Solving 3, and Mathematics Computation. The tendency for the problem solving scores to determine two mathematics factors for boys and one mathematics factor for girls was shown also by the canonical correlation analysis.

Conclusions

One explanation for the sex differences in cognitive structures as determined primarily by the problem solving scores is that girls approached problem solving more systematically; consequently, their methods for solving Problem Solving 3 paralleled their solutions to Problem Solving 2. Boys may have used established rules and algorithms when solving Problem Solving 2, but used more of a Gestalt approach for Problem Solving 3.

Both factor analysis and canonical correlation analysis indicated that performance on Problem Solving 2 was related to Verbal ability for boys and girls. Verbal ability was more highly related to Problem Solving 3 for girs than for boys. This suggested that the girls' approach to Problem Solving 3 may have been more verbal than that of boys.

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Friday, April 22, 1977
10:30 - 11:30 a.m.

Research Reporting Session
Paper Four

AN INVESTIGATION OF TRANSLATION ABILITIES AND THEIR RELATIONSHIP TO PROBLEM SOLVING

Sandra P. Clarkson
Hunter College

Purpose

The research was divided into three phases--evaluation, exploration, and instruction. The evaluation phase was designed to identify certain crucial abilities in translation, both active and passive, which discriminated among the various problem solving ability levels; the exploration phase was designed to identify certain translation strategies used by the students in solving problems; and the instruction phase was designed to determine whether providing students with instruction in the crucial, or discriminating, translational schema is sufficient for improving their problem solving performance on a paper and pencil test.

Procedures

A battery of tests compiled from scales given to the Y-population in the National Longitudinal Study of Mathematical Abilities was administered to determine mathematical and related competencies and to group students into ability levels. Passive translation ability was determined by performance on a series of paper and pencil tasks created by the investigator. Given a verbal, symbolic, or pictorial representation, students were required to recognize both valid and invalid translations. These task tests did not serve to identify whether or not a student could, in fact, translate from one representation to another--they merely tested whether or not a student could recognize a translation. To determine the active translation ability of the students, the investigator administered paper and pencil tasks requiring students to actually perform translations.

In the exploration stage students were asked to solve problems. The interviews were taped and the tapes analyzed to determine the translation strategies used.

Based on the information gained in the first phases of this study, instructional procedures were designed to provide students with the identified crucial translation variables to determine if improvement in these translation schema, assuming such improvement could be effected, would lead to corresponding improvement in problem solving, as measured on a posttest.

The subjects were students in five Algebra I classes in a small industrial town in Georgia. All classes were given the translation tasks, problem solving pretest, and a series of tests to determine other factors. Two classes were selected and, based on the preliminary analysis of the

translation tasks, were instructed in specific translation techniques. At the end of the period of instruction, the students were tested to determine which had mastered the translation techniques. All students were then given the posttest.

Analysis

Analysis prior to the instructional phase included analysis of variance and discriminant analysis to determine which of the concomitant variables (translation scores) independently discriminated among problem solving ability groups at a significant level and to determine the "best" discriminators among problem solving ability groups with the effect of several variables considered simultaneously in a single discriminant function. This information was used in designing the instructional phase. Information on other analyses performed will be available later.

Results and Conclusions

The data are in the process of being analyzed, so results are not yet available. The results and conclusions will be reported at the meeting.

Friday, April 22, 1977
12:00 - 1:00 p.m.

Panel Presentation

LEARNING OF CARDINAL AND ORDINAL NUMBERS

Martin L. Johnson
University of Maryland

Leslie P. Steffe
University of Georgia

Robert Underhill
University of Houston

During the Spring of 1975, financed by a grant from the National Science Foundation, the Georgia Center for the Study of Learning and Teaching in Mathematics sponsored a series of five research workshops covering the topics (a) teaching strategies in mathematics, (b) number and measurement concepts, (c) space and geometry concepts, (d) models for learning mathematical concepts, and (e) problem solving. From each workshop, numerous working subgroups were formed by research interest.

The cardinal-ordinal number working group was one of the subgroups formed from the number and measurement workshop. It was the expressed purpose of this group to conceptualize, design, and carry out research related to how young children learn cardinal and ordinal number ideas and how these ideas are used in their mathematical work. Within the group some members are investigating the adequacy of Piagetian models in explaining the formation of early number concepts, while others are investigating cardinal-ordinal ideas using models that differ completely from Piaget's. Teaching experiments, normative studies, and validation studies are all part of research currently being conducted.

A search of the literature reveals the need for such a research interest. Very little conclusive evidence is available about how children conceive of cardinality and ordinality. Little is found about how children use or when they become capable of using cardinal and ordinal number ideas. Furthermore, the effect of varying amount of emphasis on cardinal and ordinal number problem solving strategies has not been systematically studied. Even with this lack of information, arguments are being made that more ordinal number work should be part of the elementary curriculum. Before these arguments can be properly evaluated more research is needed.

During the past year, much interaction has taken place among the members of this working group. Much information related to learning of cardinal and ordinal numbers has been gathered. Models for the study of cardinal-ordinal number have been developed and a vast amount of research generated using some of these models. Tentative hypotheses have been formulated and much needed research identified. There is a need to share this information with the mathematics education community. Hopefully, such an exchange will stimulate and facilitate continued research in this area.

Three speakers will make 15 minute presentations. Questions will be entertained from the audience during the final 15 minute segment. It has not been determined if each presenter will prepare a paper for the session.

Friday, April 22, 1977 Round Table Discussion
12:00 - 1:00 p.m.

THE EFFECT OF STRATEGY ON 1, 2, AND 3 DIMENSIONAL TRANSFORMATION TASKS

John C. Moyer
Marquette University

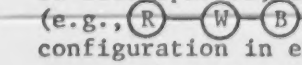
Purpose

Three dimensional lattices are very efficient for consolidating many types of data. However, their efficiency is countered by the difficulty experienced in creating, maintaining, and/or transforming three dimensional mental images. Hence, we as educators would like to determine if it is possible to improve the ability to conceptualize in three dimensions.

An interesting hypotheses with regard to this question is that the ability to conceptualize in three dimensions is related to strategies consistent with formal operational thinking (in the Piagetian sense). That is, the ability to fully take advantage of the data consolidation capabilities of three-dimensional arrays is delayed until the advent of formal operations. It is at this time that the child is first able to mentally hold certain variables constant while another is varied. Perhaps it is just such an ability that is required when working with three dimensional lattices. Piaget (1960) has found that the conservation of volume does not become truly operational until the advent of formal operational thinking. This correlation of three dimensional and formal operational thinking leads one to further hypothesize regarding a cause-effect relationship between these two variables.

As a working member of the Georgia Center for the Study of Learning and Teaching Mathematics I am doing an experiment in which adults and children of varying ages are required to mentally perform Euclidean transformations on different linear configurations of four colored spheres. The tasks are designed to determine: (1) if there are any differences between children's and adults' ability to perform mental transformations in 1, 2, or 3 dimensions; (2) what the strategies used in performing such tasks are; and (3) how the ability to perform these tasks is related to strategies consistent with formal operational thinking.

Procedures

During a video taped session each subject is shown a configuration of four spheres, colored red, white, or blue and connected by dowels (e.g., 

). The subject is then asked to find the same configuration in each of four networks, wherein it is embedded exactly once. The networks are:

Network I.D.	Number of Lattices	Type of Lattices	Total Number of Embedded Configurations
A	1	1 x 64	122
B	1	7 x 7	112
C	1	4 x 4 x 4	96
D	6	4 x 4	96

The subject must find the four embeddings (in Networks A, B, C, and D) of three such configurations randomly chosen from the 18 possible four-sphere, three-color configurations.

Analysis

For each of the 12 tasks the data recorded are:

(1) latency measurement (from initiation of the task to the subject's asserted "solution").

(2) frequency count of correct solutions.

(3) verbalized strategies as the subject looks for solutions.

The data collection is one-half finished at this time. Twenty students in the fifth grade and twenty more in the seventh grade have been tested. Still to be tested are twenty ninth graders and twenty college students. Testing will be completed by May 15, 1976. A 4 x 6 multivariate analysis of variance of the data will be performed to determine whether differences exist between the mean vectors due to groups and configurations. Further, a profile analysis will be done to determine whether differences exist between mean scores for each of the four networks for each of the four population groups.

A detailed study of the tapes is already revealing distinct strategies used by subjects in their search for solutions. A comprehensive analysis will be done after all the data have been collected.

Discussion

The consolidating effect of the 4 x 4 x 4 network reduces the number of embedded configurations as compared with the 1 x 64 network. Hence, probabilistically speaking, the latencies should be reduced (in the limit) for the three dimensional task. Of course this assumes equal mobility in performing one- and three- dimensional mental transformations. If this is not the case, rather than being reduced the latencies could very well be inflated. What is being sought here is a point in time when a child is first able to take advantage of the reduced number of embedded solutions. Will it coincide with the advent of formal operations?

Strategies are much more intricate for the solution of the problem in two dimensions than they are in either the one dimensional or three

dimensional networks presented. Hence it is expected that the 7 x 7 latencies will be much longer for the two dimensional case than the three dimensional case.

With the total number of embedded configurations equal and with basically the same strategies required in each of these two cases, a comparison of results in the two dimensional case will show if the addition of a third dimension changes the strategies and/or latencies involved.

Friday, April 22, 1977
1:30 - 2:30 p.m.

Round Table Discussion

SEX DIFFERENCES IN SPATIAL PERFORMANCE OF EIGHTH GRADE STUDENTS

Grayson H. Wheatley
Purdue University

Jane Kough
Wainwright Junior High School

Purpose

The purpose of this study was to determine the effects of experience with spatial tasks on eighth grade boys' and girls' spatial performance. Additionally, the role of Piagetian cognitive level (concrete, formal) in sex differences was examined.

Conceptual Framework

One of the most consistently appearing sex differences in cognitive skills is observed on spatial performance with males typically scoring higher than females (Maccoby and Jacklin, 1974). Studies have shown these differences exist for a variety of spatial tasks, including embedding figures, mental rotation, left-right discrimination, map-reading, rod-and-frame tests, identifying horizontality, geometric problems, and maze-learning (Harris, J. J., Hanley, C., and Best, C. T. and Boe, B. L., 1966).

These differences have been explained in terms of sex-typed interests (Hilton and Berglund, 1974), cerebral hemisphere specialization (Galin, 1975), the socialization process, and hormone differences (Maccoby and Jacklin, 1974; Harris, 1975). Investigations of hemispheric specialization in the cerebral cortex have demonstrated that the right hemisphere only is specialized for spatial tasks (Galin and Ornstein, 1975; Harris, 1975). A study on cerebral speech laterality, hand preference, and spatial ability by McGlone and Davidson (1972) indicated that the left cerebral hemisphere was dominant in verbal processing and the right cerebral hemisphere dominant in non-verbal spatial activities. Females were found to be left hemisphere dominant while males were found to be more right hemisphere dominant.

Fennema (1974) has conducted an extensive review of mathematics achievement studies which contain information on sex differences. She concludes that no sex differences exist during the elementary school years but that males tend to perform better at the high school level. This conclusion agrees with previous reviews (Suydam and Riedesel, 1969; Suydam and Weaver, 1970) and findings of the National Assessment of Educational Progress, 1975. Attempts at determining the genesis of these differences has considered subskills such as computation, arithmetic

reasoning, and problem solving with little consistent sex differences apparent. However, boys do consistently score higher than girls on spatial tests (Maccoby and Jacklin, 1974), particularly after elementary school. Thus the sex differences in mathematics achievement may be related to differences in spatial ability.

Hilton and Berglund (1974) found a close correspondence between emerging sex-typed interests and the divergence in mathematics achievement between the sexes. Thus the sex differences in mathematics achievement may result from different interest patterns of the sexes.

In this study, performance on spatial tests were compared before and after training on spatial tasks. If, in fact, spatial performance differences result from girls having less experience with spatial tasks, then significant improvement might be observed by the girls as a result of training on spatial tasks. If the differences are genetic as some have argued (Vandenberg, 1975) then little improvement would be observed.

Procedures

The subjects for this study were 121 eighth-grade pupils from a suburban-rural area Indiana junior high school. Six intact classes from an eighth-grade science class were used. The classes were grouped by the school as: two above-average classes, two average classes, and two below-average classes. One class of each of these categories was randomly assigned to the control group, and the other class assigned to the experimental group. This resulted in three classes in the experimental group and three in the control group.

A pretest consisting of the DAT Spatial Relations Test and a 20-item Shape Rotation test were given to all students.

In the DAT Spatial Relations test, the subjects

> visually constructs a three-dimensional figure from a two-dimensional pattern, remembers the three-dimensional image and matches it to perspective drawings of alternative objects, and, after locating a correct object, visualizes the rotation of the object in three-dimensional space and then matches it with other objects. (Harris, 1975)

The Shape Rotations test is a test in which the subjects select the one correct geometric figure that matches the stimulus shape which has been rotated in the plane and that has not been reflected. The posttest were Parallel forms of the pretests.

Each subject was given a formal reasoning test to determine their cognitive level in the Piagetian sense. The reasoning test consisted of eight items: propositional logic (two items), orientation tasks (two items), proportional reasoning (two items), and combinatorial thought (two items).

Following the pretest, the control group continued with their eighth grade ISCS science program. The teacher conducting the program allowed the students to progress individually and to work on lab experiments while the experimental group received training on spatial tasks. The topics studied were tangrams, polyhedra model building, pentominoes, and tessellations, with one day devoted to each. After the four treatment days, a posttest was given to both the control and experimental groups. The study was completed in six consecutive days. The experimenters did not participate in the teaching.

Analysis and Results

A 2 (treatment) x 2 (sex) x 2 (Cognitive Level) design was used to examine the main effects and interactions. While the data processing is not yet complete, some results are apparent. Female subjects scored significantly higher than male subjects on the pretest and posttest. The treatment was equally effective for both sexes. The nature of the interactions of the three variables will be reported. When data analysis are completed, other results will be available for reporting.

A surprising finding of this study was higher scores by females than males on the spatial tests. This superiority was consistently present on the pretests and posttests. This result is in contrast to the findings in previous studies summarized by Maccoby and Jacklin (1974).

Tables 1 and 2 below summarize the means and standard deviations (in parentheses) of the pre- and posttest scores.

Table 1

Pre- and Posttest Means and Standard Deviations (in parentheses) for the DAT Space Relations Test

		Male	Female
E	pre-	20.81 (10.91)	25.36 (12.55)
	post-	21.70 (10.55)	26.84 (14.43)
C	pre-	22.16 (13.83)	24.59 (12.59)
	post-	20.36 (11.77)	23.10 (14.89)

Table 2

Pre- and Posttest Means and Standard Deviations (in parentheses) for the Shape Rotation Test

		Male	Female
E	pre-	15.54 (6.45)	16.82 (4.31)
	post-	15.93 (5.32)	17.84 (2.20)
C	pre-	15.41 (5.25)	14.55 (5.60)
	post-	15.73 (5.67)	15.07 (5.23)

Some possible sources of the observed sex differences from other studies are (1) degree of urbanization (population was rural), (2) ability of the two groups (were girls more intelligent?), and (3) sex of teacher (do female teachers elicit better performance?). The results of this study will be interpreted in terms of sex-typed interests, Vandenberg's heredity hypothesis, and hemispheric specialization theory.

Friday, April 22, 1975
1:30 - 2:30 p.m.

Panel Presentation

SOVIET PROBLEM SOLVING RESEARCH

Jeremy Kilpatrick
University of Georgia

Mary Grace Kantowski
University of Florida

Sidney Rachlin
University of Georgia

James Wilson
University of Georgia

Purpose

To evaluate Soviet techniques of studying problem solving in mathematics and to stimulate the adaptation and use of these techniques, as appropriate, in American research on problem-solving processes.

Rationale

The purpose of the Problem Solving Project of the Georgia Center for the Study of Learning and Teaching Mathematics is to organize and implement "coordinated studies of the conditions for and the effects of learning and instruction of mathematics which emphasize problem solving." The Center proposes to initiate studies devoted to the identification of strategies and processes people use in solving mathematical problems and to develop clinical procedures for observing and analyzing mathematical problem-solving behavior. Most problem-solving research in the United States has dealt with the product aspect of problem solving. In the Soviet Union, however, primarily because of the social and political doctrines, researchers have not attempted to obtain objective measures of success in problem solving. Instead, they have concentrated on the dynamics of mental activity and have attempted to investigate the learning process itself. The Soviet clinical approach or "teaching experiment" attempts to "catch" processes as they develop. American researchers seeking to identify processes used in problem solving or attempting to develop clinical procedures for observing and analyzing mathematical problem-solving behavior might wish to explore the Soviet approach for possible modifications in their own studies.

To acquaint mathematics educators and teachers with directions, ideas, and accomplishments in the psychology of mathematical instruction in the Soviet Union, the School Mathematics Study Group and the Survey of Recent East European Mathematical Literature produced 14 volumes of translations known as the Soviet Studies in the Psychology of Learning and Teaching Mathematics. Jeremy Kilpatrick was one of the editors of

the Soviet Studies. Some modifications of the Soviet methodology have begun to appear in the problem solving research in the United States. Mary Grace Kantowski has employed a modification of the "teaching experiment" in a study of students' abilities to solve geometrical problems. She has also served as a consultant to other researchers exploring the possibilities of identifying student processes in solving problems by means of a "teaching experiment." The fact that the Soviet Studies are translations of research reports originally published over ten years ago has led some researchers to question the current state of the "teaching experiment" in the Soviet Union. Sidney Rachlin has been awarded a grant by the International Research Exchange Board to investigate current Soviet problem-solving research. A second question generated by the Soviet Studies is how to evaluate research that is not experimental in design and is reported qualitatively rather than quantitatively. As editor of JRME, James Wilson can respond to the question of the publishability of research that is not experimental in design and the means by which such research is to be evaluated. The introduction of the Soviet Studies has generated much interest among mathematics educators. The papers presented at the Problem Solving Workshop at the Georgia Center in the Spring of 1975 reflect an interest in the further exploration of the processes of solving problems in mathematics through modifications of the "teaching experiment". The proposed panel will explore the current applications of the "teaching experiment" in the problem-solving research in the mathematics education communities of the United States and the Soviet Union.

Suggested Procedures

After a brief introduction, Kilpatrick will discuss some of the Soviet problem-solving research that has been translated into English. The emphasis of this discussion will be on the methodology used in the Soviet research. Kantowski will then describe examples of research in this country which have been stimulated at least in part by the Soviet Studies. Next, Rachlin will explore some of the recent Soviet problem-solving research (within the last 7 years), emphasizing any shifts in methodology and possible implications for American researchers. Wilson will discuss some of the difficulties of evaluating Soviet-style research.

Friday, April 22, 1977
3:00 - 4:00 p.m.

Panel Presentation

MODELS OF MATHEMATICS LEARNING

William E. Geeslin
University of New Hampshire

Harold Mick
Virginia Polytechnic University and State University

Richard Shumway
Ohio State University

The purpose of this session is to report the work of the Research Workship: Models of Learning Mathematics. This research group is one of several organized by the Georgia Center for the Study of Learning and Teaching Mathematics during Spring, 1975. Our particular group is composed of 25 researchers from across the United States and Canada. Our goal is to stimulate research aimed at deriving models describing how children learn mathematics. In addition to conducting empirical research, we expend effort into conceptualizing theoretical models and combining/modifying existing models. Thus this research session would consist of discussions of possible models and reporting of empirical data that bears on these models. It should be noted that the empirical data presented would be secondary to the discussion of models and to suggestions for research questions to be investigated. In fact, the data presently being gathered by our group is exploratory in nature and not sufficient to reject or accept any particular model to be discussed.

It is hoped that this research session will acquaint the audience with our work while it is in the early formative stages. Additionally, the session should encourage other researchers to join us and communicate with us. It is our feeling that successful development of learning models will take a large cooperative effort.

We consider model development one of the most crucial problems in mathematics education. Models can be used in two major ways. First, a model may be used to simulate a process for the purpose of gaining an understanding of that process. Second, a model may be used to simulate a process for the purpose of predicting the result of that process. It should be noted that different models might be used for each purpose. Although the Models Research Workshop should be interested in both types of models, we must be careful not to assume that a good predicting model will lead necessarily to further understanding of the process of learning mathematics. A model that leads to understanding how children learn a particular aspect of mathematics should allow us to predict accurately the types of instruction that will produce learning. Additionally, we must be cautious in generalizing a particular model to situations other than the one in which the model is developed.

The ideal model for furthering understanding of mathematics learning would allow us to tie each combination of instructional inputs to student

learning in a cause and effect relationship. This ideal model would not only describe learning outcomes produced by instruction, but also allow us to view the learning process itself. That is, the simulation could be stopped at any point in time so that we could observe how the student "transforms" the instruction into learning.

A good prediction model would be of great use in instructional decisions. Nonetheless, if this model does not further our understanding of how children learn, we might be required to have a multitude of models to cover the numerous instructional environments we face. As the number of models increases, the usefulness of these models in practical classroom teaching decreases. Numerous models would almost certainly produce contradictions in determining the "ideal" instructional practice. A model used solely to predict whether or not successful learning occurs ideally would be a simpler model. This model need not imply causation. A model for predicting whether a given child learns successfully would be superior to a model that predicts only the proportions of students learning successfully. In the practical situation one is looking for a simple model that requires no more information than the ordinary classroom teacher has available.

We have identified four basic manners in which present models were developed. The four varieties are: instructional models, logical models, linguistic models, and developmental (psychological) models. Each type implies a particular model of learning. At present most models deal with only a portion of the student's learning behaviors. Thus the session will begin with a short introduction to models in general. Then each presenter will discuss an example of one of the four types of models: instructional, logical, linguistic, and developmental.

Friday, April 22, 1977
4:30 - 5:30 p.m.

Research Reporting Session
Paper One

AN EVALUATION OF AN NSF-CCSS COMPUTER EXTENDED MATHEMATICS PROJECT

Edwin D. Andersen
University of Minnesota

David C. Johnson
University of Minnesota

Purpose

The University of Minnesota and the Minneapolis Public Schools planned an in-service program that consisted of an intensive summer institute for 58 teachers and a follow-up academic year program of seminars and service activities. This research was designed to evaluate the effect of this training on the behavior of the teachers and the achievement of students the following year.

This study attempted to answer the following research questions:

1. As a result of the summer institute was there an increase in the use of the computer for instruction by the participating teachers?

2. As a result of this program was there an impact on the performance of students on a problem-solving test in each of the grades seven through twelve?

3. As a result of this program, was there a difference in the amount of use of the computer in problem-solving activities by students of participating teachers as compared to students of non-participating teachers?

Procedures

The summer instructional program was broken into several sections: Elementary, Seventh, Eighth, Algebra, Geometry, Advanced Mathematics, and Chemistry. Six instructors serviced these groups. The emphasis of the program was to examine existing computer literature, try out the computer activities and write curriculum outlines pointing out where these activities should be used in the curriculum.

The academic year phase consisted of seminars offered each quarter at the University of Minnesota and the service activities of two, half-time staff members. The seminars were designed to implement the outlines developed in the summer institute. These outlines were discussed, computer lessons were designed and evaluated and computer project materials were written. The service activities consisted of regular visits to the

summer institute participants, demonstrative teaching, resource help at teacher meetings and assistance in the selection of materials.

To answer question one, a pre-institute interview schedule was developed. The investigator interviewed each summer institute participant and determined the extent of computer utilization in instruction the previous years. In the year following the summer institute, these teachers were asked to keep logs of their computer activity. The investigator visited these teachers on a regular basis. He collected the logs and assisted the teachers with any questions.

The results for senior high teachers show a small increase (from 252 to 288) in the number of days the computer was used as a tool of instruction from 1970-71 to 1971-72. The first year, three-fourths of the use was by one teacher, the second year the use was well distributed among the seventeen teachers. There was an increase in the number of students doing problem-solving from 379 to 682. Our interpretation is that this is an important increase in computer usage in instruction.

Results for junior high teachers (from 104 to 280) show approximately a doubling in the number of days of computer use in instruction. The elementary teachers had no use in 1970-71 because terminals were not available in their schools. Terminals were available in 1971-72 and were used 109 days. The four chemistry teacher results (from 5 to 22 days) show a large increase in usage of the computer for instruction.

To measure question two, related to student behavior, a problem-solving abilities test previously developed in related research at the University of Minnesota by Thomas Foster was modified for this evaluation. A junior high version and a senior high version were developed by selecting appropriate items from the Foster instrument. A separate evaluation was done at each grade level 7-12. This test was designed to measure problem-solving abilities related to computer experiences. All sections taught by institute teachers were tested and a random sample of non-institute teacher sections was tested. A fall test was given as a pretest and the same test given in the spring as a posttest. The fall test, along with ability measures such as DAT scores and reading scores, were used in a regression analysis with the spring test score as the dependent variable. This regression analysis statistical procedure was developed by David C. Johnson in his research at the University of Minnesota. A similar procedure was used in the NLSMA study.

A regression equation of the form $Y = a + b_1X_1 + b_2X_2 + b_3X_3$ was written for each classification of the students. This model assumes a linear relationship between each of the independent variables and the dependent variable. In the statistical analysis the students were classified as institute (their teachers attended the summer institute), or non-institute (their teachers did not attend the institute). A F test was used to check if one regression equation would fit all classifications. If there were distinct regression equations, the regression coefficients were used to derive adjusted mean values for the dependent variable. A 90% confidence interval was computed for the adjusted means If there was no overlap of the confidence intervals, the investigator decided that the adjusted means were different.

The results show that the institute students had higher adjusted means than the non-institute students at grade levels 7, 8, 10, and 12. Another classification concerned the academic year seminar status of the student's teacher. If the student's teacher earned at least one seminar credit, he was classified seminar. If the student's teacher did not earn credit in the seminars, he was classified non-seminar. The seminar classification had higher adjusted means at grade levels 7 and 8. The non-seminar classification had a higher adjusted mean at grade level 12. All other grade levels had overlapping confidence intervals on their adjusted means.

To evaluate question three, all students on the spring problem-solving abilities test were asked to classify the number of problems they had attempted to solve by writing computer programs. Three levels were identified as: 1..little (PS1), 2..moderate (PS2) and 3..considerable (PS3) computer problem-solving work. These classifications were used in the regression analysis and show that at grade levels 7, 8, 10 and 12 computer problem-solving levels PS2 and PS3 had higher adjusted means on the problem-solving abilities test.

The computer problem-solving data was used in a breakdown by classification as institute and non-institute. Results show that in general the institute sections had higher proportions of students in computer problem-solving categories PS2 and PS3 than did non-institute sections. The seminar versus non-seminar classification showed no clear trend.

Conclusions

In summary, this study suggests that the NSF-CCSS Inservice Education Project on Computer-Extended Mathematics did change the teacher's classroom behavior in the direction of more computer instructional activitiy. This study suggests that the project had a positive effect on the student performance on a problem-solving test. This study suggests that the project had a positive effect on the amount of computer problem-solving the students attempted the following year.

Friday, April 22, 1977
4:30 - 5:30 p.m.

Research Reporting Session
Paper Two

SMALL GROUP-DISCOVERY LEARNING IN CALCULUS: ITS EFFECT UPON ACHIEVEMENT AND ATTITUDES

Christian R. Hirsch
Western Michigan Universtiy

Sister Mary Catherine Brechting
Aquinas College

Purpose

The primary purpose of this study was to investigate the relative efficacy of a small group-discovery method and the conventional lecture-discussion method in promoting concept attainment, skill acquisition, and favorable attitudes toward mathematics in an introductory calculus course.

Conceptual Framework

While it is generally recognized that one of the major pedagogical goals of mathematics instruction should be the active participation of students in the learning process, this goal is seldom or only incidentally achieved in most under-graduate courses in mathematics. That increased attention should be paid to this goal in collegiate instruction has been urged by Cummins (1960), Polya (1963), Moise (1965), Stein (1972), and Halmos (1975) among others.

Closely related with this view is the hypothesis that "for efficient learning, the learner should discover by himself as large a fraction of the material to be learned as feasible under the given circumstances." (Polya, p. 608) Though considerable research has been conducted to assess the effectiveness of discovery teaching and learning, the results of these studies suggest that the effectiveness of this approach is still an open question (Begle, 1969). Moreover, most of this research has been limited to mathematical instruction at the elementary and secondary levels.

Finally, though there appears to be an increasing trend toward individualization of introductory courses; there is, on the other hand, a growing body of research which suggests that learning is a social process in which the learner becomes a partner in a shared activity. (cf., MacPherson, 1972; Steiner, 1972)

The integration of the above views of learning into a small group-discovery approach to calculus instruction was first formulated and tested by Davidson (1971). Though Davidson provided both a theoretical base and existence proof for this instructional strategy, the selectivity of the subjects in the experimental class (twelve volunteers with A or B grades in high school mathematics) left open the question of the efficacy of such

an approach. The present study was then an attempt to provide additional empirical data regarding the viability of this method in introductory calculus.

Procedures

The subjects were 46 students enrolled in two afternoon sections of the introductory calculus course at Western Michigan University during the winter semester of 1975. The subjects included both males and females of freshman, sophomore, junior, and senior status. One section (N = 21) was designated to use the experimental materials and the other section (N = 25) served as a control group.

The pre-calculus competence of the subjects in both groups was measured prior to instruction with a test consisting of 35 multiple-choice items selected from the Cooperative Mathematics Tests (ETS) Algebra II, Algebra III, and Analytic Geometry. Students in both groups were also given the Aiken-Dreger Revised Mathematics Attitude Scale to assess their attitudes toward mathematics on entering the course.

The groups were then given instruction in two different modes and utilizing two different kinds of instructional materials for the duration of one semester. Both groups met for 50 minutes 4 times per week. In the experimental treatment, subjects were organized in four groups with 4 members per group and a fifth group with 5 members. The classroom activity was a social process taking place in the small groups, with their learning directed by guide sheets based upon materials originally developed by Davidson, but revised and supplemented to be consistent with the content presented to the control group. The students, with some guidance from the instructor, formulated definitions, constructed examples and counterexamples, discovered and proved many of theorems of calculus, and developed techniques for solving various classes of problems. The control group was taught by an experienced professor whose instructional methods were judged to be representative of the conventional lecture-discussion approach.

Upon completion of the experiment, subjects in both treatments were readministered the attitude scale and were given a comprehensive test consisting of 36 multiple-choice items (odd numbered items involving primarily manipulative skills and even numbered items measuring understanding of concepts) selected from the Cooperative Mathematics Tests Calculus, Parts I and II and Vervoort's (1970) Calculus Test. Experimental subjects also responded to an open-ended questionnaire regarding their learning environment.

Analysis and Results

Analysis of covariance, with student pretest scores as the covariate, was applied to the data about the two treatments on the total posttest, skills subtest, and concepts subtest. Post-treatment attitude scores were also adjusted using analysis of covariance with pre-treatment attitude scores as the covariate. Prior to each analysis the assumption of

homogeneity of regression was tested and in each case homogeneous regression was clearly tenable.

The ANCOVA on total posttest scores resulted in the rejection ($p < .02$) of the null hypothesis of no effects due to instructional treatment. The adjusted mean for the experimental group was 20.65 and the adjusted mean score for the control group was 17.21.

Similarly, the ANCOVA on skills subtest scores resulted in the rejection ($p \leq .01$) of the null hypothesis of no treatment effects. The experimental group had an adjusted mean of 13.02 and the control group's adjusted mean was 10.35.

The null hypothesis concerning differences in concept attainment was not rejected ($p = .2807$); the adjusted means for the experimental and control groups were 7.64 and 6.87 respectively. A further analysis of the results indicated a significant interaction ($p < .02$) between pre-calculus achievement and treatment in the case of those items dealing with differentiation. The regression analysis suggested that pupils who scored above 14 on the pretest had a better conceptual understanding of differentiation if their instruction involved small group-discovery learning, whereas those pupils who scored 14 or below did better under a conventional format.

With respect to attitudes toward mathematics, no significant ($p = .4327$) difference between treatments was found. The adjusted mean for the experimental group was 53.66, while the adjusted mean score for the control group was 51.24.

Results of the open-ended questionnaire indicated that experimental subjects' perception of their learning environment was highly favorable.

Conclusions

In general, subject to the conditions of this investigation, the results of this study indicate that the use of small group-discovery learning in an introductory calculus course can be an effective means of improving student achievement, particularly with respect to the acquisition of manipulative skills. The results regarding concept attainment are not quite as encouraging. However, an item analysis yielded a mean index of difficulty of .61 for the concepts subtest as compared with .36 for the skills subtest. This suggests that the concepts subtest was too difficult for the subjects involved and this may in turn have resulted in random errors of measurement which reduced the possibility of finding a more significant difference between the treatments.

Finally, the results in the affective domain indicate that even though most students had a favorable attitude toward the small group-discovery method of instruction, the effect of this method on improvement of attitudes toward mathematics was minimal.

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Friday, April 22, 1977
4:30 - 5:30 p.m.

Research Reporting Session
Paper Three

THE UTILIZATION OF ECHOIC CODES BY VISUALLY HANDICAPPED IN MATHEMATICAL LEARNING: AN EXPLORATORY INVESTIGATION

John C. del Regato
The University of Oregon

William E. Lamon
The University of Oregon

Purpose

The major purpose of this investigation was to explore the effects of specialized acoustical instruction with an appropriate sensory aid on the mathematical learning of young blind students. A sensory device with its associated instructional methodology provided a tonal system for arithmetic symbolism of ten digits, the additon process, and equality. The degree of difficulty encountered in discrimination and identification within one of two experimental acoustical systems was described for each participant. Due to the lack of a suitable standardized instrument, a tonal mathematical performance assessment was developed to measure each participant's ability in acoustically presented arithmetic tasks.

Conceptual Framework

The literature suggests a need for developing instructional systems employing learning aids that would better facilitate mental arithmetic for the blind. Research literature on learning through listening, focusing on verbal processes, reveals significant increases in listening ability prior to sixth grade level. The studies on sensory compensation and compressed speech suggest that existing cognitive deficits among blind children are at least partially attributable to the lack of an alternate input channel which can match the visual processing rate. A review of pertinent research on absolute judgements of acoustic variables led to the conclusion that the human capacity concept of a fixed upper limit on absolute judgment accuracy is an erroneous generalization. Evidently stimulus dimensionality as well as practice contribute to the identification task.

The paucity of educational research regarding an instructional system utilizing tonal configurations to communicate mathematical concepts to blind learners led the investigators to examine the plausibility of such an approach. Three facets of the resulting investigation are readily discernible: (1) a tonal model for teaching numeral identification, place value, and addition of whole numbers incorporating an researcher-designed tonal mathematics performance assessment; (2) an instructional procedure emphasizing the utilization of tonal embodiments; and (3) the use of a case-study data collecting format.

Procedures

Twelve non-randomly selected visually handicapped youth ranging in age from four to eighteen received a profile analysis of their individual achievements in digit recognition, place value, and addition of whole numbers, all presented in a tonal modality. Most of the participants displayed additional learning handicaps other than those resulting from visual deficits. Instructional sessions for each individual participant lasted from 15 to 30 minutes, averaging 27 sessions over a period of nine weeks. The participants were all students at a residential school for the blind.

Conclusions

The results of this study are learner specific, yet on the basis of twelve case-studies, it was concluded that this investigation:

1. Presented evidence which provided objective verification of the value of acoustical stimulation programs in arithmetic symbolism for blind learners;

2. Identified learner specific perseverative errors evidenced with respect to tonal digit identification;

3. Illustrated that substantial gains at the tonal symbolic associative level was not sufficient to effect an overall arithmetic achievement as measured by the researcher-designed diagnostic-achievement test;

4. Developed A Tonal Mathematics Performance Assessment (ATMPA) which identified the mastery level in acoustical arithmetic of each participant;

5. Uncovered a statistically significant correlation between diminishing visual integrity and tonal arithmetic achievement;

6. Suggests the possibility of enhancing arithmetic achievement for visually handicapped with additional learning disabilities by presenting appropriate acoustical instruction to supplement tactual, visual, or verbal modalities; and

7. Contributes to the literature a detailed account of acoustical arithmetic instruction which might be used in future educational programming or research.

The investigation concludes with several remarks concerning its implications for education and future research and development. Recommendations for application, research, and development are proposed.

Friday, April 22, 1977
4:30 - 5:30 p.m.

Research Reporting Session
Paper Four

COGNITIVE DEVELOPMENT AND DIFFICULTY IN SOLVING WORD PROBLEMS IN MATHEMATICS

Janet Hudson Caldwell
University of Pennsylvania

Purpose

The process of solving mathematical word problems can be divided into two general phases: translation and computation. In the translation phase, the student translates a verbal statement of the problem into a mathematical expression or equation. In the computation phase, he performs the operations on the expression or equation which are necessary to find the answer. The kind of situation described in the problem statement may affect the difficulty of translating it into mathematical expressions and, consequently, the overall difficulty of the problem.

The purpose of this study is to compare the relative difficulties of four types of word problems - abstract factual, abstract hypothetical, concrete factual, and concrete hypothetical - with respect to populations at different developmental stages.

An abstract word problem is defined as a word problem involving a situation which describes abstract or symbolic objects only, while a concrete word problem is defined as one describing a real situation dealing with real objects. For example, a problem about digits in a number would be abstract, while a problem about baseballs would be concrete.

A factual problem is defined to be one which simply describes a situation. A hypothetical problem is defined to be one which not only describes a situation, but also describes a possible change in the situation. This change does not really occur within the context of the problem. In solving the hypothetical problem, the problem-solver must consider not only the situation which occurs within the context of the problem, but also the described alteration which does not occur. Four examples are given below:

1. There is a certain given number. Three more than twice this given number is equal to fifteen. What is the value of the given number?

   Abstract and factual. No change is described.

2. There is a certain number. If this number were four more than twice as large, it would be equal to eighteen. What is the number?

   Abstract and hypothetical. The number is not really four more than twice as large.

3. Susan has some dolls. Jane has five more than twice as many, so she has seventeen dolls. How many dolls does Susan have?

Concrete and factual. No change is described.

4. Susan has some dolls. If she had four more than twice as many, she would have fourteen dolls. How many dolls does Susan really have?

Concrete and hypothetical. Susan does not really have four more than twice as many dolls.

The comparative difficulty of these types of word problems may be considered from two viewpoints. A developmental model would suggest that, for younger subjects, concrete problems and factual problems should be less difficult than abstract problems and hypothetical problems, respectively, while there should be no necessary difference in difficulty for older subjects. An information processing model would suggest that abstract problems are less difficult than concrete problems and that factual problems are less difficult than hypothetical problems. The translation of an abstract problem into a mathematical expression is relatively direct, since the expression and the problem solution are themselves abstract, whereas translating a concrete problem into an abstract mathematical expression requires additional processing. Similarly, decoding a hypothetical problem is a more complex process than decoding a factual problem.

Procedures

Data from the administration of word problem tests in arithmetic and elementary algebra to approximately 1200 subjects in grades four through twelve will be presented. The results of parallel computational skills tests administered to the same subjects will also be presented. The word problem tests consist of five to seven sets of four problems; each set is similar to the examples given previously. The problems within each set are isomorphic with respect to length, syntactic structure, vocabulary level, underlying algebraic equation, sequencing of information, and magnitudes, signs and types of numbers (fractions, integers, decimals, etc.). The problems differ with respect to the two variables previously described: each is either concrete or abstract and each is either factual or hypothetical. The four combinations constitute each set of four problems. The basic experimental design is a multifactorial analysis of variance with repeated measures on the concrete-abstract and factual-hypothetical factors. Other factors to be considered are grade, sex, order ot test administration and whether the subject passes or fails the computational skills test. The variables involving sex, grade and order of testing are primarily experimental controls included to reduce error variance.

In order to isolate experimental effects more clearly, the computational skills test variable has been introduced, classifying the subjects on the basis of their ability to perform the calculations needed to solve the problems. Since the intent of this study is to focus on the translation phase of solving word problems, incorrect answers which are obtained as a result of computational errors contribute error to the estimate of

experimental effect. By considering the subject's competence with regard to the computation phase, this error can be reduced.

Outcomes with respect to the following null hypotheses will be reported:

1. There is no significant difference between the number of abstract problems solved and the number of concrete problems solved.

2. There is no significant difference between the number of factual problems solved and the number of hypothetical problems solved.

3. There are no significant grade differences.

4. There are no significant sex differences.

5. There are no significant differences with respect to the order of test administration.

The results of this study will either affirm or contradict current practices in teaching how to solve word problems, as well as indicating whether developmental or information processing models are preferable. Most problems presently included in elementary school texts are concrete and factual, abstract problems presently appear in the junior high texts and are found more frequently in high school texts, and a very few hypothetical problems appear randomly throughout the grades. Concrete factual problems may indeed be easier for younger students, confirming the current predominance of such problems in elementary school texts. It is possible, however, that abstract ones might be easier and thus should be included at earlier grade levels. It is also possible that older subjects need additional practice in solving concrete problems. The results of this study will indicate which types of problems are most difficult for students at different grade levels under current instructional conditions.

Saturday, April 23, 1977
9:30 - 10:30 a.m.

Round Table Discussion

A STUDY OF A TEACHING CENTER PROGRAM AND TEACHING PERFORMANCE IN MATH

Violet C. Weiss
Brevard Inservice Teaching Center

Purpose

The Brevard Inservice Teaching Center is an alternative inservice program designed to fulfill student needs by improving teacher competencies.

The terminal objective of the project is that students and teachers participating in the program will demonstrate positive changes (significant at $p < .05$) in achievement of specific goals which are:

Student	Teacher
1. To improve mathematics skills.	1. To improve ability to teach mathematics effectively.
2. To improve attitudes toward mathematics.	2. To improve attitude toward mathematics.
	3. To improve attitudes toward ability to teach mathematics adequately (self-concept).

This project has been funded through an ESEA Title III for innovation programs. The planning year 1972 - 73 was followed by three operational years.

The population in the three-year study was 109 K - 5 teachers and 3,270 K - 5 pupils in the target area of Central Brevard County. The schools involved in the research project were randomly-assigned as experimental and control schools.

Procedures

The Brevard Inservice Teaching Center is a program in which practical and theoretical inservice education combine to meet the needs of teachers and students. The key factors in the model program are as follows:

1. Provision of a systematic, competency-based program.

2. Use of a team approach in which the participating teacher is matched with a specially-trained Staff Teacher.

3. Bringing the participating teacher's own students to the place of inservice where the teacher may actually learn and implement simultaneously through the daily teaching process with her own students.

During pre-planning, participating teachers are teamed with Project Lead Teachers, and the teachers' needs are diagnosed using two methods. The teacher completes a self-diagnostic instrument which identifies the individualized inservice modules (LAPs) best suited to meet specific needs. Assisted by the Senior Teacher for Staff Development, the participating teacher completes the LAP activities which are designed to move from theory to application in the classroom.

Simultaneously, through observation and student placement testing, the Lead Teacher determines the teachers' needs in relation to her particular classroom and develops a plan of action. This needs assessment enables the Lead Teacher to guide the participating teacher in planning instructions through the use of objectives, activities and materials from the Brevard County LAMP. This teaming also assists the participating teacher in selecting and implementing the appropriate methods and materials that are presented in workshops.

The participating teacher's own classroom is the instructional setting where the knowledge accumulated from the inservice plan is applied. The Lead Teacher is able to lend support through joint planning, teaching and evaluating performance.

Through the use of a video-tape system and very structured peer panel procedures, the participating teacher evaluates the strengths and weaknesses of the teaching process. In subsequent planning sessions the Lead Teacher is able to follow up with specific recommendations on ideas generated during the peer panel.

The research design is based on pre- and posttesting equivalent groups achieved by stratified random selection of schools. The experimental schools received treatment at the Center while the control schools received no treatment. Student Objectives were tested with the following tests:

1. Math Skills (Grades 1 - 5) - Comprehensive Test of Basic Skills (CTBS)

   Math Skills (Kindergarten) - Test of Basic Experience (TOBE)

2. Math Attitudes - Dutton Attitude Test

3. Self-Concept (Grades K - 2)
   Yeatts - Bently, I Feel - Me Feel

   (Grades 3 - 5)
   Ira Gordon, How I See Myself

Teacher Objectives were tested with the following tests:

1. Ability to Teach Mathematics Effectively - Florida Climate and Control System (FLACCS) and Teacher Practices Observation Record (TPOR). Both FLACCS and TPOR consist of direct observation of the teacher in the classroom with a checklist of observable behaviors divided into specific categories.

2. Attitude Toward Mathematics - Dutton Attitude and Specific categories of the FLACCS and TPOR.

3. Attitude Toward Ability to Teach Mathematics Adequately - (Self-Concept) The Teacher Self-Report Inventory (TSRI) developed by Purkey and Persons and specific categories of the FLACCS and TPOR.

Results

For Math Competency (the composite of Competencies 7 and 12), the control variables of grade and grade squared were significant, with all control variables accounting for twenty-six percent (26%) variance, and both program and the interaction of program with student pretest math mean were significant ($p < .05$), together accounting for an additional fourteen percent (14%) of the variance (equivalent to a correlation of .37).

TABLE A

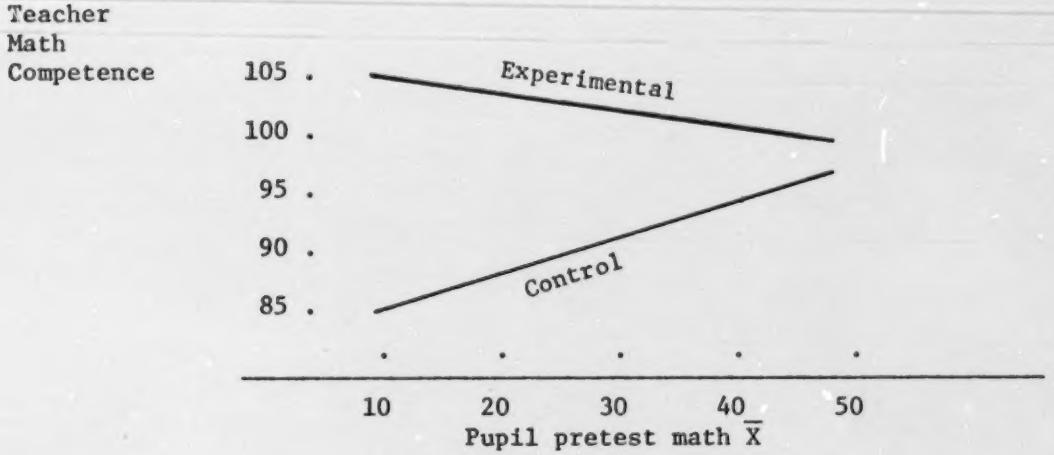
Teacher Change Associated with Experimental Program[a]
(Restricted Model)

Outcome Variable	Control Variable	F	%V	Program Variable	F	%V	Step
Math Competency (07+12)	Grade	15.74***		Program	10.33**		6-
	Grade(Sq)	13.26***	26	Program X Pre-Math $\bar{X}$	4.16	14	
Manages Classroom (07)	Grade	15.82***		Program	9.98**		6-
	Grade(Sq)	12.85***	25	Program X Pre-Math $\bar{X}$	4.37*	14	
Math Teaching (12)	Grade	8.42**		Program	4.82*	7	5
	Grade(Sq)	7.65**	20				

[a] N=60
* P < .05
** P < .01
*** P < .001

In order to identify the nature of the interaction between program and student pretest math mean, the data were platted (Figure A).

The Interaction of Program with Pupil Pre-Test Math Mean in Determining Teacher Math Teaching Competence



The figure indicates that adjusted posttest scores for experimental teachers were higher than for control teachers at all levels of pupil pretest math means. However, the major difference appears to be the difference between the teaching skills of experimental and control teachers with low ability pupils. The program was able to produce for experimental teachers with low pretest math means, teaching skill which was as good as the skill of control teachers with high pupil pretest math means.

The program has been successful in increasing the skills of teachers in teaching low ability pupils so that the difference between experimental and control teachers working with low ability pupils was greater than one and a half standard deviations.

For the self-concept measure, grade level and grade squared were the significant control variables, accounting for twenty percent (20%) variance, along with the other control variables. The variables of program and the interaction of program with pupil pretest math mean were significant ($p < .05$), but in this case program became significant only after the interaction had entered.

TABLE B

Teacher Change Associated with Experimental Program [a]
(Restricted Model)

Outcome Variable	Control Variable	F	%V	Program Variable	F	%V	Step
Self-Concept (4+6)	Pre Self-Concept	9.99**		Program	4.66*[b]		6-
	Grade	4.41		Program X			
	Grade (Sq)	4.11	20	Pre-Math $\overline{X}$	4.49*	7	

[a] N=60

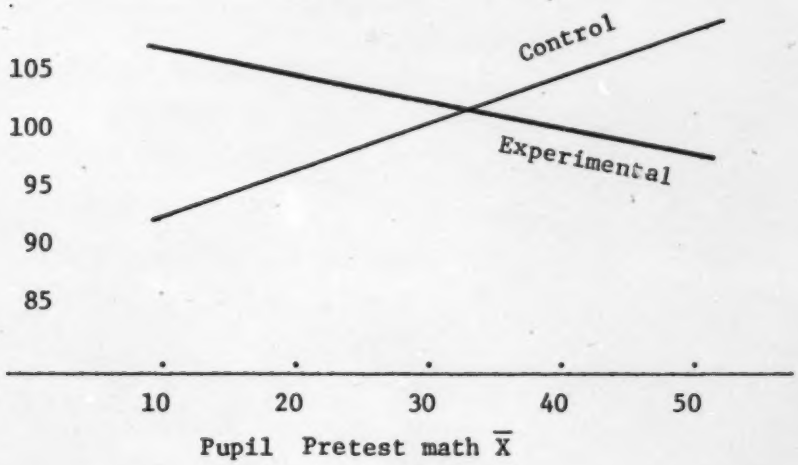
[b] Becomes significant only when the interaction enters.

* P<.05
** P<.01
***P<.001

Interaction is plotted below.

FIGURE B

The Interaction of Program with Pupil Pre-Test Math Mean in Determining Teacher Self-Concept



Experimental teachers had a more favorable self-concept than control teachers when working with pupils with low pretest math means, whereas control teachers had a more favorable self-concept working with pupils with high pretest math means. In short, the effect of the program was highly dependent on the pretest ability level of the pupils for the outcome measure of teacher self-concept.

Statistical evidence that the student objective was attained is shown in the summary report on the analysis of student data for the first two years of operation

TABLE C

Analysis of Student Math - 1973 - 74

Group	Pre-Test Mean E	Pre-Test Mean C	Post-Test Mean E	Post-Test Mean C	t value	P
Kindergarten	15.13	15.27	17.29	14.75	4.218	.005*
First Grade	15.06	15.46	35.80	30.68	3.029	.005*
Second Grade	16.79	20.60	35.80	29.20	1.368	NS(.15*)
		($p < .005$)				

TABLE D

Analysis of Student Math - 1974 - 75

Group	Pre-Test Mean E	Pre-Test Mean C	Post-Test Mean E	Post-Test Mean C	F value	P
Third Grade	26.59	38.40	54.76	54.57	6.745	.05*
Fourth Grade	54.628	51.117	73.806	65.403	0.097	N.S.
Fifth Grade	43.300	46.485	47.871	58.969	1.045	N.S.

* Differences were in favor of the experimental group.

In accordance with the evaluation design, the independent probabilities for the two years were combined by Dr. Robert S. Soar, Evaluation Consultant, to obtain an aggregate probability.[1] This method bases a new test of significance on the combined data from all of the groups.

TABLE E

Pooled Probablilities
Across Grade Levels

Grade	P	1-P[a]	X^2	df
K	.005		10.60	2
1	.005		10.60	2
2	.15		3.79	2
3	.05		5.99	2
4	.99[b]		.02	2
5	.20[c]	.80	.45	2
		X^2 =	31.45	12

P<.01

a. When results are obtained counter to prediction, 1-P is used instead of P.

b. The F was small enough that a tabled value near it could not be found. Accordingly, a probability of .99 was used as an extreme conservative value.

c. This grade level produced results counter to prediction.

Despite the overall significance (P<.01) of the difference in favor of the experimental program, the results strongly suggest an interaction with grade level in which the program is most effective at the lower grades. However, probabilities from the six groups of K - 5 students currently being post-tested remain to be pooled.

1 Gordon, Mordecai H., Loveland, Edward H., and Cureton, Edward E., "An Extended Table of Chi-Square for Two Degrees of Freedom, for Use in Combining Probabilities From Independent Samples." Psychometrika, Vol. 17, No. 3, September, 1952.

Student and teacher data are continuing to be collected and analyzed. Final results will not be available until June 30, but will be incorporated in the report when presented.

Conclusion

The Brevard Inservice Teaching Center program has improved teacher effectiveness and teacher attitude toward ability to teach mathematics adequately with increased pupil achievement in mathematics. No measurable differences were found in student self-concept nor in student and teacher attitude toward mathematics.

Saturday, April 23, 1977
9:30 - 10:30 a.m.

Research Reporting Session
Paper One

THE ROLE OF PICTURES IN CHILDREN'S PERCEPTION OF MATHEMATICAL RELATIONSHIPS

Patricia F. Campbell
Purdue University

Purpose

An examination of current primary mathematics textbooks reveals that one of their means of communicating mathematical concepts to children is by pictures, in conjunction with written numeral statements. These pictures are intended to serve as a reference to assist the students in solving a related problem. Often the pictures themselves may supply sufficient information so as to provide the solution. For example, a dynamic picture may clearly suggest addition or subraction and allow the child to determine the answer by simply counting the objects in the picture. It is hoped that the children are simultaneously making a transfer from the dynamic situation illustrated in the picture to the abstract number sentence accompanying the picture. However, it is initially necessary for the children to relate the characters depicted in the pictures and to perceive the action portrayed before they can associate the pictures with the addition or subtraction of whole numbers.

The purpose of this study was to investigate whether there is a relationship between first grade children's ability to tell a story about a picture or a sequence of three pictures and their ability to describe the picture(s) by a number sentence. The children's stories were analyzed in terms of the degree to which the stories assimilated the characteristics of the picture(s) and indicated perception of motion. Further, the artistic variables characterizing the pictures were controlled so as to provide information concerning which types of illustration best facilitated interpretation of the pictures and perception of mathematical relationships.

Conceptual Framework

Research concerning children's perception of pictures indicates that children initially notice only discrete items depicted in a picture and gradually develop the ability to describe a picture in terms of overt action and to perceive relationships between the illustrated characters (Amen, 1941; Carpenter, 1964; Travers, 1969). Motion is one of the more difficult representations for children to perceive in static pictures; perception of motion may be dependent on the children's own experience with change and the degree to which the artistic conventions depicting motion within the illustration are realistic to the child (Schnall, 1968; Travers, 1969; Friedman & Stevenson, 1975).

Analysis of video-taped interviews recorded by the Project for the Mathematical Development of Children during 1974-75 suggests that first

graders offer varying interpretations of pictures depicting mathematical relationships and, at times, may become confused by the pictures. In order to interpret the mathematical relationship depicted in a picture, it seemed necessary for the child to assimilate or relate the characters in the picture and, in dynamic illustrations, to perceive motion.

Examination of current primary mathematics textbooks revealed that most dynamic illustrations were single pictures, with sequences of related pictures presented primarily during the introduction of the concepts of addition and subtraction. The illustrations used both human figures depicted in simple settings (realistic drawings) and personified animals or inanimate objects in settings characteristic of humans (stylistic drawings).

Procedures

Ten textbook illustrations were chosen as models; five depicting addition events and five depicting subtraction events. For each model, two similar sequences of drawings were drafted--a realistic sequence of three pictures and a stylistic sequence of three pictures. Each picture was 5" by 8".

The sequences of drawings depicting addition were in the following form:

Picture A: n characters depicted with a simple background.

Picture B: The same n characters as in Picture A were presented being joined by m other characters. The two groups of characters were disjoint. The same background as in Picture A was utilized.

Picture C: n + m characters depicted in one group. The same background as in Picture A was utilized.

The subtraction sequences had a similar format. The values of m and n were such that m, $n \leq 5$ and $m + n \leq 7$.

A set of single pictures was obtained by using only Picture B from each sequence.

In order to permit an analysis of the effects of the form of the drawing (realistic versus stylistic), the number of pictures in an item (sequence of three drawings versus a single drawing) and response condition (choosing a number sentence [force] versus writing a number sentence [no force]), an 8 x 2 design was utilized.

Ninety-six first graders, randomly selected from five first grade classes within two elementary schools in Tallahassee, Florida were randomly assigned to each of the 16 cells. Each subject was tested individually. Upon presentation of a single picture or a sequence of three pictures, the child was requested to tell a story that went with the picture(s). The child was then asked either to choose a number sentence card or to write

a number sentence for the picture(s). This procedure was repeated for each of the ten items presented to the child. All testing was tape recorded.

Analysis

A procedure was defined in order to assign a score for each story and number sentence response (0, 1, 2, 3, or 4). Each subject's story was also coded as to whether it revealed perception of motion (0 or 1). Summing these scores over the 10 items, each student had three total scores: a characterization of the level of assimilation revealed in his stories (0-40), his overall perception of motion (0-10), and his number sentence responses (0-40).

The Kruskal-Wallis test (a non-parametric, distribution-free, analysis of variance) was used to determine the effect of drawing style, of the response condition, and of the number of pictures on the assimilation level within the stories, on the perception of motion, and on number sentence responses. Further, the responses of the subjects assigned to the single picture cells and to sequence-single picture cells were compared on the last five items only. On these last five items, these subjects were all viewing comparable single pictures. This was done in order to determine whether viewing and describing sequences was a learning experience which assisted the subjects when viewing single pictures.

Kendall correlation coefficients were also computed to assess the relationship between the ability to tell a story about a picture or sequence and the ability to describe the picture(s) by a number sentence.

Results

The Kruskal-Wallis tests revealed that neither the drawing style (realistic versus stylistic) nor the number of pictures (single, sequence or a combination of single and sequence pictures) had a significant effect on either the level of assimilation within the stories, the perception of motion or the number sentence responses. Analysis of the response condition (force versus no force) revealed a significant difference favoring the force condition on number sentence responses. A significant difference was also noted in the level of assimilation on the last five items between the single and the sequence-single treatments. Subjects who initially responded to five sequences had a significantly higher level of assimilation on the remaining five single pictures than subjects who responded only to the single pictures.

No evidence of a correlation between the ability to tell a story about a picture or sequence and the ability to describe that picture or sequence by a number sentence was noted.

Conclusions

Based upon the results determined by this study, some conclusions are tentatively suggested.

The responses of first graders in this study revealed no differences due to the drawing style. Children at this age seem to interpret stylistic pictures of personified animals or objects with an ease equivalent to their interpretation of realistic pictures of children. Use of both types of illustrations may provide variety without adversely affecting understanding.

Although no significant differences overall were noted due to the number of pictures, initially viewing and interpreting sequences did provide a learning experience to significantly effect the interpretation of single pictures. It is recommended that illustrated sequences be presented within the mathematics curriculum and that they should be presented prior to single pictures.

The illustrations within this study utilized a particular format regarding placement of characters, spacing, detail, and depth perception. Readers are cautioned not to generalize these results to other types of pictures.

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Saturday, April 23, 1977
9:30 - 10:30 a.m.

Research Reporting Session
Paper Two

A LONGITUDINAL STUDY: SCORES ON PIAGETIAN AREA TASKS AS PREDICTORS OF ACHIEVEMENT IN MATHEMATICS OVER A FOUR-YEAR PERIOD

Thalia Taloumis
Brooklyn College of the City University of New York

Purpose

There has been no longitudinal research that has investigated the relationship of performance on Piagetian area tasks and performance on standardized measures over an interval of several years. Most of the research on the prediction of achievement in mathematics from performance on Piagetian tasks has been conducted at first grade level or within a span of one year of that grade (Steffe, 1966; LeBlanc, 1968; Nelson, 1969; Kaufman and Kaufman, 1972; and Smith, 1973). Other predicting research studies have involved other grade levels (Goldschmid, 1967; Kaminsky, 1970; Silliphant and Cox, 1972; and Rohr, 1973), but still cover an interval of a year.

The report of the longitudinal research is based on the results of the original dissertation study published in the Journal for Research in Mathematics Education (Taloumis, 1975). In the latter study five Piagetian tasks (conservation and measurement) were administered in one sitting to each of 168 middle-class children, ages six years five months to nine years four months. Statistical procedures included an analysis of variance, a Pearson Product-moment correlation, and multiple-regression correlation for subgroups. The findings indicated (1) that whichever group of area tasks (area conservation tasks or area measurement tasks) was presented second, that group of tasks resulted in higher scores; (2) that there was significant correlation between area conservation scores and area measurement scores; (3) that boys and girls did not score significantly differently on area conservation tasks and area measurement tasks; and (4) that scores in area conservation tasks and area measurment tasks showed no significant difference in amount of increase from grade to grade.

Conceptual Framework

There has been a significant revolution that has taken place in school mathematics curricula over the past fifteen years. Area measurement is found in programs as early as the second or third grade and involve diagrams of regions on pegboards or graph paper, and the sectioning of regions into units. Piaget's models for investigating cognitive development, being mathematically based, are found adapted in programs such as the Nuffield Project booklets (1967) and books on modern mathematics curricula such as Freedom to Learn (Biggs, 1969). Some complete early childhood curricula have been based on Piagetian activities such as Dr. Celia Stendler's "Kit" (1970). The question arises as to whether mathematical achievement can be predicted once the level of cognitive development,

or at least, those with low or high cognitive development can be identified for an individualized placement in a mathematics program.

Procedures

The current longitudinal study has investigated three major aspects relevant to the relationship of performance on Piagetian area tasks and other measures of performance in mathematics:

1. the relationship of scores of Piagetian area conservation and area measurement tasks administered to first, second, and third graders (Taloumis, 1973) for predicting teacher grades in mathematics and Stanford Achievement Test scores in mathematics for each of four succeeding years;

2. the relationship between scores on the Piagetian area tasks and the SRA Primary Mental Abilities Test for predicting performance in mathematics achievement (i.e., do either scores contribute to the predicition of later progress in mathematics achievement? If so, do they predict equally as well or is one a better predictor than the other?);

3. the relationship between any two variables: Piagetian area task scores, SRA Primary Mental Abilities Tests, Stanford Achievement Tests, and teacher grades.

Analysis and Results

Statistical procedures included correlations and multiple regressions for two I.Q. tests, teacher marks, Stanford Achievement Tests (SAT), conservation, and measurement scores. Findings indicated that (with a high frequency) (1) conservation and measurement scores predicted teacher marks and SAT scores for group two (initially assessed in second grade) for each of four succeeding years; (2) Otis-Lennon Mental Ability Tests predicted teacher marks and SAT scores for all groups for each of four succeeding years; and (3) given four ordered predictors (I.Q. tests and Piagetian area tasks cores) for teacher marks and SAT scores, conservation was the most frequently significant predictor for group two, while Otis-Lennon I.W. scores was for group one and three.

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Saturday, April 23, 1977
9:30 - 10:30 a.m.

Research Reporting Session
Paper Three

THE EFFECTS OF THREE INSTRUCTIONAL SETTINGS ON THE LEARNING OF GEOMETRIC CONCEPTS BY ELEMENTARY SCHOOL CHILDREN

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University of North Dakota

Purpose

The purpose of this research was to investigate the effects of three instructional settings on the learning of selected geometric concepts by third grade subjects. The goals for teaching the selected geometric concepts were: (1) to develop a basis for the learning of geometric principles, and (2) to develop an ability to transfer these geometric concepts to problem solving situations relating to these selected concepts.

The selected concepts which were included in this research were: point, line, line segment, triangle, square, pentagon, ray, angle, congruence of lines, congruence of angles, polygon, cube, tetrahedron, face, vertex, edge, diagonal, side, and endpoint.

Procedures

This study was conducted in a Minneapolis-St. Paul suburban school system at the third grade level during the school year. The sample consisted of 169 Ss (82 Ss from School One and 87 Ss from School Two). Ss and teachers were randomly assigned to Treatment Groups.

Special units for the three experimental Treatments in each school were prepared by the researcher. The experiment was designed so that each S in the study received a programmed instructional unit. The programmed instructional units for each of the three Treatments had the same objectives and developed the same concepts. The only difference in the programmed instructional units was the instructional setting.

The materials in Treatment W were written using the traditional textbook-worksheet apprach where every practice page was a paper and pencil worksheet. The materials included explanations of each concept as well as questions to be answered as the Ss worked through the programmed instruction.

The Ss in each school randomly assigned to Treatment M studied the same concepts as the Ss assigned to Treatment W. In Treatment M the practice activities and mode of presentation of the selected concepts was changed to manipulatives. The manipulatives were: geoboard, georuler, and paper folding.

The Ss in each school randomly assigned to Treatment S studied the same concepts as the Ss assigned to Treatments W and M. Treatment S differed from Treatments W and M in mode of presentation and practice of the selected concepts. In Treatment S, practice consisted of activities involving demonstration and manipulation of geometric solids.

Scores on the ITBS were used as the blocking variable in each school. Due to the variability of the ITBS scores, each school had different ranges for high, middle, and low.

The pre-treatment measure ITBS was given to each third grade student in early fall semester. The Post Treatment measures were constructed by the researcher and served as the criterion measure for testing the propositions of the study. The Post Treatment measures were a Posttest (11th day of experiment), a Retention Test I (21st day), and a Retention Test II (31st day.) Each test was divided into two parts, Part One and Part Two, where Part One measured goal One and Part Two measured goal Two.

The study investigated 36 major propositions. Because of the pairwise comparisons, three tests, and three ability levels, only one type of each proposition will be listed in detail. Type One: there is no significant difference between the group of third grade Ss receiving Treatment W and the group of Ss receiving Treatment M in mean performance on the Posttest designed to measure their achievement on selected geometric concepts and their ability to transfer these selected concepts to solve related geometric problems. Type Two: There is no significant difference between the group of third grade Ss receiving Treatment M and the group of Ss receiving Treatment S in mean performance on the Posttest (part Two) designed to measure their ability to transfer these selected concepts to solve related geometric problems.

Analysis

For each test (Post, Retention I and Retention II) of this p x q factorial experiment within each school, an ANOVA procedure (Winer, 1971) was used to test the significance of differences between mean scores. Contrasts were constructed between pairs of scores and the Bonferroni t-statistics (Miller, 1966) was used to calculate simultaneous confidence intervals for all established propositions (See Table 1).

Results

Since the observed interaction F ratios were all smaller than the .05 critical value, the data does not reject the assumption of zero interaction. The tests on main effects for the treatment factor has an observed F ratio that is significant in seven cases. Hence the treatment means can be considered to be significantly different at the .05 level for these seven cases.

Bonferroni t-statistics. Propositions were established to examine the treatment effects. Simultaneous $1 - \alpha(.95)$ Bonferroni confidence

intervals were calculated for these contrasts and are reported. (See Table 1.)

TABLE 1

BONFERRONI CONFIDENCE INTERVALS

PROPOSITION	SCHOOL ONE Bonf. Conf. Int.	SCHOOL TWO Bonf. Conf. Int.
Post Test (Part One and Part Two) Total (Measuring Goal One and Goal Two)		
W vs M	(-1.45,2.21) Not Rej	(-1.08,2.18) Not Rej
W vs S	(-4.12,-.42) Rej (S>W)	(-2.46, .84) Not Rej
M vs S	(-4.53,-.77) Rej (S>M)	(-2.99, .27) Not Rej
Within High (Measuring Goal One and Goal Two)		
W vs M	(-2.96,3.06) Not Rej	(-2.29,3.39) Not Rej
W vs S	(-4.12,2.30) Not Rej	(-3.36,2.52) Not Rej
M vs S	(-4.07,2.15) Not Rej	(-3.74,1.80) Not Rej
Within Low (Measuring Goal One and Goal Two)		
W vs M	(-1.46,5.44) Not Rej	(-1.51,4.51) Not Rej
W vs S	(-6.96,-.10) Rej (S>W)	(-4.64,1.38) Not Rej
M vs S	(-9.06,-1.94) Rej (S>M)	(-6.14,-.12) Rej (S>M)
Retention Test I (Part One and Part Two) Total (Measuring Goal One and Goal Two)		
W vs M	(- .84,2.52) Not Rej	(-1.55,1.57) Not Rej
W vs S	(-2.89, .51) Not Rej	(-1.66,1.50) Not Rej
M vs S	(-3.76,0.30) Rej (S>M)	(-1.65,1.47) Not Rej
Within High (Measuring Goal One and Goal Two)		
W vs M	(-2.95,2.59) Not Rej	(-3.69,1.79) Not Rej
W vs S	(-3.99,1.89) Not Rej	(-3.37.2.21) Not Rej
M vs S	(-3.71,1.97) Not Rej	(-2.27,3.01) Not Rej
Within Low (Measuring Goal One and Goal Two)		
W vs M	(-2.87,3.45) Not Rej	(-2.24,3.50) Not Rej
W vs S	(-5.04,1.28) Not Rej	(-2.74,3.00) Not Rej
M vs S	(-5.41,1.07) Not Rej	(-3.37,2.37) Not Rej

Rentention Test II (Part One and Part Two)
Total (Measuring Goal One and Goal Two)

W vs M	(-1.31.1.51) Not Rej	(- .76,2.54) Not Rej
W vs S	(-3.06,-.20) Rej (S>W)	(-3.37,-.01) Rej (S>W)
M vs S	(-2.99,-.07) Rej (S>M)	(-4.23,-.93) Rej (S>M)

Within High (Measuring Goal One and Goal Two)

W vs M	(-3.19,1.41) Not Rej	(-2.24,3.50) Not Rej
W vs S	(-3.65,1.25) Not Rej	(-3.94,1.98) Not Rej
M vs S	(-2.69,2.07) Not Rej	(-4.40,1.18) Not Rej

Within Low (Measuring Goal One and Goal Two)

W vs M	(-3.93,1.33) Not Rej	(-1.04,5.04) Not Rej
W vs S	(-5.53,-.27) Rej (S>W)	(-5.04,1.04) Not Rej
M vs S	(-4.32,1.12) Not Rej	(-7.04,-.96) Rej (S>M)

Post Test (Part Two)(Measuring Goal Two)

W vs M	(- .99, .39) Not Rej	(- .59, .59) Not Rej
W vs S	(-1.61,-.23) Rej (S>W)	(- .67, .51) Not Rej
M vs S	(-1.36, .12) Not Rej	(- .67, .51) Not Rej

Retention Test I (Part Two)(Measuring Goal Two)

W vs M	(-1.33, .89) Not Rej	(-1.04, .74) Not Rej
W vs S	(-1.99, .29) Not Rej	(- .52,1.30) Not Rej
M vs S	(-1.77, .51) Not Rej	(- .37,1.43) Not Rej

Retention Test II (Part Two)(Measuring Goal Two)

W vs M	(-1.48, .54) Not Rej	(- .69,1.39) Not Rej
W vs S	(-2.50,-.42) Rej (S>W)	(-2.02, .16) Not Rej
M vs S	(-2.03, .05) Not Rej	(-2.32,-.24) Rej (S>M)

Conclusions

The experimental means show a directional trend with Treatment S superior to Treatment W and Treatment W superior to Treatment M. The directional trend is given strength by the Bonferroni tests. The .95 Bonferroni confidence intervals found significant differences only in favor of Treatment S. In School One for the 36 propositions tested, six were significant in favor of Treatment S over Treatment W with four significant in favor of Treatment S over Treatment M. In School Two for the 36 propositions tested, one was significant in favor of Treatment S over Treatment M. For the nine propositions for School One on all tests measuring the ability to transfer the geometric concepts to problem solving situations relating to these selected concepts there were two propositions rejected at the .05 level and both found Treatment S superior to Treatment W. For the same nine in School Two, there was one proposition rejected at the .05 level and it found Treatment S superior to Treatment M.

For the nine propositions posed in each school on all tests within the high ability level, there were no significant differences at the .05 level. For the nine propositions posed in each school on all tests within the low ability level there were three rejected at the .05 level in School One, two found Treatment S superior to Treatment W and one found Treatment S superior to Treatment M, and in School Two, there were two propositions rejected at the .05 level both found Treatment S superior to Treatment M.

The following implications are presented by the researcher. There is a definite directional trend in the means on every post treatment measure in each school (Treatment S was superior). This supports the claim of some mathematics educators, i.e. Ss are better able to learn geometric concepts when the concepts are presented using geometric solids in a multi-embodiment presentation. After perusing the propositions with the low ability level in School One, Treatment S was superior to Treatment W in two propositions. Hence this research showed that for low ability Ss, the solid multi-embodiment approach (Treatment S) is an appropriate mode of presentation.

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Saturday, April 23, 1977
9:30 - 10:30 a.m.

Research Reporting Session
Paper Four

THE RELATIVE EFFECTIVENESS OF NON-GRADED AND GRADED INSTRUCTION AS TO MATHEMATICS ABILITY AND ACHIEVEMENT AND THE ATTITUDES OF ELEMENTARY SCHOOL CHILDREN TOWARD MATHEMATICS

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Purpose

This study was designed to determine the relative effectiveness of a non-graded approach to elementary mathematics instruction and a traditional graded approach by comparing an experimental group of pupils who had had six years of non-graded schooling with three control groups who had had six years of graded schooling.

Although educators continue to evidence considerable interest in the non-graded approach to elementary mathematics instruction, research support for non-graded mathematics instruction is generally lacking. Relatively few well-designed studies have been effected to evaluate non-graded mathematics instruction, and most longitudinal studies have been executed for only three years or less. Whereas no one study can answer all of the queries posed regarding non-graded mathematics instruction, the present study has produced evidence as to the effectiveness of non-graded mathematics instruction in terms of pupil aptitude, achievement, and attitudes.

Procedures

The non-graded mathematics instruction investigated involved complete curricula planning to accommodate the non-graded philosophy. The pupils comprising the experimental group were together for six years in the Chesapeake Demonstration School, a non-graded elementary school funded over a four-year period (1967-70) with grants totaling more than $1,300,000 under Title III, ESEA. This school has employed, from its inception, a locally-devised systems approach to mathematics instruction. The progress of each pupil in mathematics is carefully logged as specific behavioral objectives are mastered. Modular scheduling, team teaching, teacher aides, independent study, and intensive pupil involvement are hallmarks of the school. The two most notable characteristics are the absence of any grade designations whatsoever and the absence of marks and report cards. Furthermore, homework is not assigned on a regular basis since it, like daily activities, is planned to meet individual learner needs. This study involves the first group of pupils to complete six years of schooling under these conditions.

The experimental group consisted of 52 pupils who had completed six years of schooling at the Chesapeake Demonstration School, a non-graded elementary school in Chesapeake, Virginia. These pupils were each 12 years old when the data were gathered.

The first of three control groups, hereafter to be known as Control Group A, was drawn from the total population of approximately 400 pupils at the Western Branch Junior High School, where 55 pupils (in addition to the experimental group) were found who had attended Chesapeake graded schools for six years. Comparisons of mean scores on the Metropolitan Readiness Test (Durost, Bixler, Hildreth, Lund, Wright, and Stone, 1959), administered upon school entry in 1967, and on the Kuhlmann-Anderson Tests (Kuhlmann & Anderson, 1960), administered in 1968, revealed no significant differences.

Matched-pairs procedures were used to select the second control group, hereafter known as Control Group B, from among 384 pupils identified as possible control subjects (that is -- they had attended graded schools in Chesapeake). The criteria used for matching were: six years of schooling in the Chesapeake Schools, sex, age, race, socioeconomic level, readiness scores, and second-grade intelligence test scores. Forty-eight pairs of pupils meeting all criteria were found. No control subjects could be found to match four experimental subjects; therefore, there were 48 matched pairs rather than 52.

The third group was selected through a stratified random sampling technique. Of 384 control pupils available, 197 were selected, hereafter to be known as Control Group C.

Certain control subjects were selected for more than one of the control groups.

Experimental and control subjects were compared as to mathematics aptitude (Cognitive Abilities Test-Quantitative and Non-Verbal subtests), mathematics achievement (SRA Achievement Series Mathematics Concepts and Mathematics Computations) and attitude toward mathematics (Semantic Differential Technique). Trained technicians administered the test in each of several testing stations to which both experimental and control subjects were randomly assigned. Explicit written instructions, discussed in several training sessions, were supplied all persons administering the tests; and carefully controlled testing protocols were followed throughout the testing procedures. Each test was scored twice, by first one trained scorer and then another, to insure accuracy of scoring.

Analysis

The data were subjected to an analysis of variance in order to investigate main effects and interactions between variables. Blocking was performed on intelligence scores and socioeconomic levels in order to reduce subject variance.

Pupils in the experimental group scored significantly higher on the Non-Verbal subtest of the Cognitive Abilities Test (Experimental: M=65.24;

Control Group C: M=60.77, F= 4.54, df=1/137, p<.05) and on the Mathematics Concepts subtest of the SRA Achievement Series (Experimental: M= 26.78; Control Group C: M=22.99, F=7.44, df=1/137, p<.01) than did pupils in Control Group C.

In the Control Group A comparisons, a significant main effect occurred with more affluent pupils scoring significantly higher on the Kuhlmann-Anderson Test (Higher SES: M=105.23; Lower SES: M=98.87, F=8.22, df=1/83, p <.01), on the Non-Verbal subtest of the Cognitive Abilities Test (Higher SES: M=65.24; Lower SES: M=57.90, F=6.28, df=1/83, p <.01), on the Mathematics Concepts subtest of the SRA Achievement Series (Higher SES: M=28.28; Lower SES: M=22.66, F=13.23, df=1/83, p <.001) and on the Mathematics Computations subtest of the SRA Achievement Series (Higher SES: M=25.60; Lower SES: M=20.51, F=6.60, df=1/83, p<.01). The only significant interaction occurred between socioeconomic level and treatment group on the Quantitative subtest of the Cognitive Abilities Test (F=4.43, df=1.83, p<.001), with higher socioeconomic-level pupils scoring approximately the same in both graded and non-graded mathematics instructional programs and lower-socioeconomic level pupils scoring higher in the non-graded school.

No other effects in this analysis reached significance.

Saturday, April 23, 1977
11:00 - 12:00 a.m.

Panel Presentation

RESEARCH IN PROBLEM SOLVING PROCESSES

Philip Smith
Southern Connecticut State College

Nick Branca
The Pennsylvania State University

Dorothy Goldberg
Kean College of New Jersey

Howard Kellogg
Brooklyn College

John Lucas
University of Wisconsin-Oshkosh

In recent years, research in mathematical problem solving has been moving toward greater emphasis on the solution process (that is, on the set of behaviors or activities that direct the search for a solution) in addition to recognizing the correct solution. Such a movement suggests the need for a change from the purely statistical analysis of numbers of correct responses to a more subjective coding of protocols of problem solutions. Justification for the new emphasis arises because:

(1) correct processes may result in incorrect solutions due to minor computational errors;

(2) at the other extreme, guessing could result in correct responses without any cognitive activity to support the choice of response;

(3) the finding of some regularities in the solution processes could result in implications for research in problem solving.

Although the clinical methodology which was used extensively by Soviet researchers, among others, offers the best possibilities for analysis of problem solving protocols, problems of choosing a symbolic notation to represent the processes and problems of communication to other researchers the meaning of the chosen symbols exist.

A team of researchers has recently been engaged in a series of studies in which a common coding scheme, refined by the entire group, is being used. The studies are clinical in nature, are longitudinal, and are conducted across age levels. Certain common problems as well as problems most applicable to the individual age groups are used in the studies.

The purpose of the panel's proposed presentation will be:

(1) to discuss the consortium model for research as it has worked in one particular case (Schulman suggests that a carefully planned program of research is necessary if answers to questions about problem solving are to be found);

(2) to summarize the research currently being done on problem solving processes by the members of the consortium;

(3) to discuss the need for scoring a solution to include process as well as product;

(4) to introduce the process-coding scheme being used, to describe the elements of the coding scheme together with the processes they represent, and to show how these processes are related to Polya's heuristics;

(5) to discuss the future work of the consortium and the implications of its work for other researchers;

(6) to induce other problem solving researchers to make explicit certain commonalities among their own work and that of the consortium.

An outline of the panel's presentation follows:

I. Introduction
II. Discussion of current research and coding scheme
III. Playing of (video or audio) tape protocol of an actual problem solution and simultaneous coding of the protocol
IV. Discussion of implications of consortium's work for future research
V. Questions and discussion

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